Towards the Ultimate APL-TOE (⋆)

Gérard A. Langlet
CEA/DSM/DRECAM/SCM, Lab. de Recherche en Informatique
C.E. Saclay, F-91191-Gif sur Yvette Cedex, France
Facsimilé: (33) 1 69 08 79 63

Keywords:
Integrals, Fractals, Symmetry, T.O.E, Topology, Dynamical systems, Parity, Holography, Genetics, *APL*, Automata, Chaos, Fibonacci, Propagation, Algorithms, Periodic systems, Binary algebra.

Dedicated to John A. Wheeler & Kenneth E. Iverson

Abstract

This paper presents the results of more than 10 years of transdisciplinary work. The initial idea was: can the laws of Nature also been found or rebuilt, independently from theoretical research in Physics (on elementary particles and matter in general), also in the field of Computer Science i.e. Information Processing? Pressing a lemon reveals its juice and stones; if one "tortures" matter, the components of it (first electrons, neutrons and protons, then quarks and gluons at a lesser order of magnitude) may be detected. What will appear if one tries to compress algorithms instead of atoms? *APL* seemed to be the ideal candidate for such a systematical investigation that led to some intriguing results which first proved to be indeed strongly connected with the conventional laws of Physics, then might enlighten in a new way many apparently-independent observations and studies, in a variety of fields such as neural networks, natural-language and signal processing, fractal geometry, Biology and Genetics inter alia.

⋆ **Theory Of Everything** (in theoretical physics).

Definitions and Notation

Vectors, Sequences and *Masses*

If **V** is any Boolean vector, the corresponding Boolean Sequence **S** is its mirror image ⌽**V**. So, **V** is also given by ⌽**S**.

When **V** is the same as **S** , then the sequence is a palindrome.

The *total mass m* of a vector or sequence is the number of its bits ⍴**V**.

The *weighing mass mw* of a vector or sequence is the number of its 1-bits +/**V**.

The *hidden mass mh* of a vector or sequence is the number of its 0-bits +/~**V** or +/**1**≠**V**.

Boolean Matrices

Let **G** be the following Boolean matrix (named [main] *geniton* from now on). The other matrices **Gv Gh Gd** are the 3 mirror images of **G** :

G	Gv ⌽G	Gh ⊖G	Gd ⌽⊖G (or ⊖⌽G)
1 1	1 1	1 0	0 1
1 0	0 1	1 1	1 1

The *antigeniton* and its images are the Boolean negations:

0 0	0 0	0 1	1 0
0 1	1 0	0 0	0 0

In the Boolean field, any square matrix, filled with 0's except the main diagonal (filled with 1's) will be called **U** (for Unit matrix). **Z** is the all-0 matrix.

So, the corresponding **U** and **Z** as well as the anti-arrays, given by Boolean negation, are :

U	**Z**	anti-**U** (also ⌽**U** and ⊖U)	anti-**Z**
1 0	0 0	0 1	1 1
0 1	0 0	1 0	1 1

Operations in the Boolean field

The Binary Matrix Product (**BMP**) is performed by ≠.∧ which is the equivalent, in the Boolean field, of +.× in the numeric field :

≠ (XOR) is PLUS MODULO 2 while
∧ (AND) is TIMES MODULO 2.

The Binary Matrix Inverse (**BMI**) is performed on a matrix **M**, giving **IM** , (in some simple cases by ⎕CT<|⌹M), so that the result of **M BMP IM** as well as the one of **IM BMP M** returns **U**. Matrix **IM** , i.e. **BMI M**, may not exist.

The Growth Operation (**GO**) is performed on any matrix **M**, by **M∧⊂M** then removing the enclosing boxes (autosimilar growth).

The Binary Vector Integral (**BVI**) or Integral Sequence is obtained on any vector **V** by ≠**V** . The last item of the **BVI** is the Binary Scalar Integral (**BSI**), given by ≠/**V** .

Topology

All vectors and matrices may be presented under shapes other than the conventional row-column display, and with notations other than the usual binary couple 1 & 0 (charges, spins, sex): + - ↑↓ **X Y** or other antagonist couples can be used, e.g. the 2-geniton can be presented as :

```
                                          X
+ +    - -    ↑ ↑    ↓ ↓    X X    or   X   X
+ -    - +    ↑ ↓    ↓ ↑    X Y             Y
```

Nevertheless, it will always be considered as a Boolean matrix, except otherwise stated.

Postulatum

The Main Mechanism of the Topologic Universe is Parity Integration.

1 being the odd parity and **0** the even parity, (i.e. an integer MODULO 2), any Boolean sequence or vector or matrix can be considered as the parities of any array **A** of integers with the same shape. The parity of **A** is 2|**A** , or, rather, 0≠2|**A**.

The integral sequence of **S** , after parity integration, is given by ≠**S** , as defined previously.

Correlatively, the binary scalar integral of **S** is ≠/**S** or ¯1↑≠**S** .

The Elementary Bit

1 is the elementary weighing bit. It is either a scalar, or a vector (palindrome) or a matrix with shape 1 1 . It is then its own matrix inverse and mirror images. As a sequence, it is also its own parity integral. Let us say that this elementary bit contains all the energy **E** of the topological universe.

The Elementary Sequence

The elementary sequence is the elementary bit followed by what is not itself, noted **0** (the elementary hidden mass). The integral sequence of **1 0** is then **1 1** , the integral sequence of which is **1 0** i.e. the elementary sequence itself.

The Pariton

For any sequence **S**, the *pariton* **P** is defined as the matrix which contains, in its successive rows, all the successive sequence integrals (or vector parity integrals) of the sequence. Then, for the elementary sequence **1 0** , the corresponding pariton is :

1 1
1 0 i.e. the hereabove-defined geniton **G**.

Any square pariton with **n** rows and **n** columns will be named a **n**-pariton from now on. In this paper, **n** always corresponds to a power of 2. The main geniton is a 2-geniton i.e. the 2-pariton of the elementary (ontological) couple.

Cycles

Cycle *c* will be defined as the periodicity of the successive integrals of a sequence. It is easy to demonstrate, by recurrence, that, for any given sequence **S** with total mass *m*, the same sequence **S** is obtained as the *c*th integral sequence, i.e. after *c* iterations, *c* being the power of 2 not lesser than *m*.

For the elementary bit, *c* equals 1, and, for the elementary sequence, *c* equals 2. For *m*∊3 4 then *c* is 4, while for *m*∊5 6 7 8 then *c* is 8, etc...

Majority

The *majority* **MAJ** of an array is 1 if the array contains more 1's than 0's; conversely, it equals 0 if it contains more 0's than 1's or as many 0's as 1's. Then, the majority of the elementary bit is 1, i.e. the same as the majority of **G**. The majority of a unit matrix **U** is 0. The majority of the 2-antigenitons is also 0.

If **G** or rather ⊂**G** is seen from the distance, the individual bits of **G** may not be perceived. Let us define perception by the fact that an object (a topologic galaxy or star) is seen when its majority is 1.

The pariton of 1 0 0 0 is (iterating ≠\ 4 times) **GG**:

1 1 1 1	
1 0 1 0	the majority of which is 1.
1 1 0 0	
1 0 0 0	Such a matrix is a *4-pariton* and also a geniton, the *4-geniton*.

GG may be considered as composed of three **G**'s and one **Z**:

G G	
G Z	which is another notation for **GG**, using the couple **G** and **Z**.

This property of autosimilarity is then applicable, by recurrence, to larger matrices such as **GGG**, the pariton of 8↑1 :

1 1 1 1 1 1 1 1	which is : **GG GG** or :	**G G G G**
1 0 1 0 1 0 1 0	**GG ZZ**	**G G G Z**
1 1 0 0 1 1 0 0		**G G Z Z**
1 0 0 0 1 0 0 0		**G Z Z Z**
1 1 1 1 0 0 0 0		
1 0 1 0 0 0 0 0	Such a matrix is a *8-pariton* and	
1 1 0 0 0 0 0 0	also the *8-geniton*.	
1 0 0 0 0 0 0 0		

Its majority is now 0. But the majority of the sub-cells **GG** is still 1 (the majority of **ZZ** being 0).

Topologically, all these structures are recursive: The elementary bit itself may be considered as a majority of hidden not-perceived entities which are perhaps **G** itself, or **GG**. When we come closer to such a structure, using a topological spacecraft, a magnifier (microscope or telescope), we see smaller embedded structures which may exhibit similar patterns at any scale. This is typical of <u>fractals</u>. In fact, the pariton of 1 followed by as many 0's as one wishes, is always autosimilar at any scale. It is square if *m* is a power of 2, because *m*≡*c* as we have seen. If **1** is called black and **0** white, this construct gives the image of **Fig. 1** on a video display.

This is indeed a *Sierpinski* triangle, a well-known **fractal** shape. As shown in [LA1,2,3], such large *hypergenitons* may be obtained either by *c* successive integrations of *c*↑1 or iterating **G**←**G**∧⊂**G** at will and removing the boxes or enclosures at every level; this can be done in *APL2* by formatting, see [LA1].

This last process gives rise to an autosimilar exponential growth of the primitive topological universe. If the boxes, not removed, are drawn as *membranes*, one obtains nice pictures such as the ones of [LA2], which mimic cell growth, every elementary cell containing automatically its *genetic patrimony*, i.e. **G** itself.

The sequence from which any pariton is built up by parity integration always corresponds to the last row of the pariton. In the case of all genitons, the weighing mass of the last row is always 1. The E energy which is contained in the 1-geniton is now shared by *c*×*c* positions. The weighing mass of genitons is always a power of 3 : 1 3 9 27 81 243 etc...

The *cost* of a "job" producing a pariton from a bit sequence with some weighing mass *mw* and total mass *c*, will be proportional:

a) to the number of quantified parity swaps:

≠\ 0 0	produces 0 0	(no inversion)
≠\ 0 1	produces 0 1	(no inversion)
≠\ 1 0	produces 1 1	(inversion)
≠\ 1 1	produces 1 0	(inversion)

so to the number of 1's i.e. to M the weighing mass *mw* - 1 (the first 1 is never swapped);

b) to the number of individual parities to be loaded into the computer bit-registers for checking, i.e. to the length of the sequence, i.e. to the total mass *c*, for each **BVI**, i.e. to the number of columns of the pariton (in fact *c* - 1 couples);

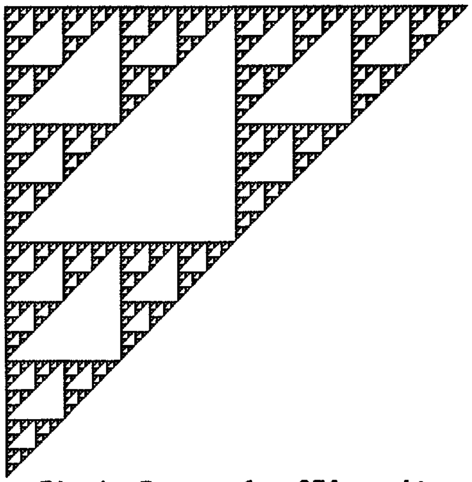
c) to the number of iterations of ≠\ i.e. to *c* again (*c* cycles: the number of rows of the pariton);

i.e. altogether to M×*c*⋆2 (M#*mw* for large *mw*).

Isn't the energy necessary to create any new entity the *total energy* of this entity?

Bur. Fich.Édite Arrêt Options

IMAGE GENITON 256 FACTOTUM Jeudi 12 mars 1992, 3h 48



Comments:

The binary sequence S for a N-geniton is N↑1 .

Then, for all genitons, which are symmetric matrices, C the last column (the cognitive transform - see further on) is also S i.e. N↑1 .

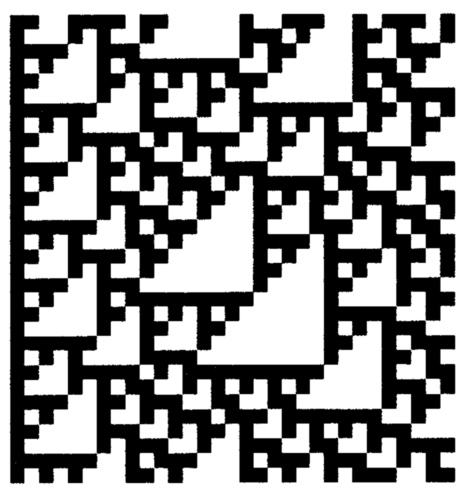
The second diagonal H (the helix transform - see further on) is N⍴1 .

Fig.1. Image of a 256-geniton, cf. [LA5] APL.68000 on the ATARI ST

APL.68000 NiveauII

```
0 1 1 0 0 1 1 1 1 0 1 1 0 0 0 0 0 1 0 0 1 1 1 0 0 1 0 1 1 1 0 1
0 1 0 0 0 1 0 1 0 0 1 0 0 0 0 0 0 1 1 1 0 1 0 0 0 1 1 0 1 0 0 1
0 1 1 1 1 0 0 1 1 1 0 0 0 0 0 0 0 1 0 1 1 0 0 0 0 1 0 0 1 1 1 0
0 1 0 1 0 0 0 1 0 1 1 1 1 1 1 1 1 0 0 1 0 0 0 0 0 1 1 1 0 1 0 0
0 1 1 0 0 0 0 1 1 0 1 0 1 0 1 0 1 1 1 0 0 0 0 0 0 1 0 1 1 0 0 0
0 1 0 0 0 0 0 1 0 0 1 1 0 0 1 1 0 1 0 0 0 0 0 0 0 1 1 0 1 1 1 1
0 1 1 1 1 1 1 0 0 0 1 0 0 0 1 0 0 1 1 1 1 1 1 1 1 0 1 1 0 1 0 1
0 1 0 1 0 1 0 0 0 0 1 1 1 1 0 0 0 1 0 1 0 1 0 1 0 0 1 0 0 1 1 0
0 1 1 0 0 1 1 1 1 1 0 1 0 1 1 1 1 0 0 1 1 0 0 1 1 1 0 0 0 1 0 0
0 1 0 0 0 1 0 1 0 1 1 0 0 1 0 1 0 0 0 1 0 0 0 1 0 1 1 1 1 0 0 0
0 1 1 1 1 0 0 1 1 0 1 1 1 0 0 1 1 1 1 0 0 0 0 1 1 0 1 0 1 1 1 1
0 1 0 1 0 0 0 1 0 0 1 0 1 1 1 0 1 0 1 1 1 1 1 0 1 1 0 0 1 0 1 0
0 1 1 0 0 0 0 1 1 1 0 0 1 0 1 1 0 0 1 0 1 0 1 1 0 1 1 1 0 0 1 1
0 1 0 0 0 0 0 1 0 1 1 1 0 0 1 0 0 0 1 1 0 0 1 0 0 1 0 1 1 1 0 1
0 1 1 1 1 1 1 0 0 1 0 1 1 1 0 0 0 0 1 0 0 0 1 1 1 0 0 1 0 1 1 0
0 1 0 1 0 1 0 0 0 1 1 0 1 0 0 0 0 0 1 1 1 1 0 1 0 0 0 1 1 0 1 1
0 1 1 0 0 1 1 1 1 0 1 1 0 0 0 0 0 0 1 0 1 0 0 1 1 1 1 0 1 1 0 1
0 1 0 0 0 1 0 1 0 0 1 0 0 0 0 0 0 0 1 1 0 0 0 1 0 1 0 0 1 0 0 1
0 1 1 1 1 0 0 1 1 1 0 0 0 0 0 0 0 0 1 0 0 0 0 1 1 0 0 0 1 1 1 0
0 1 0 1 0 0 0 1 0 1 1 1 1 1 1 1 1 1 0 0 0 0 0 1 0 0 0 0 1 0 1 1
0 1 1 0 0 0 0 1 1 0 1 0 1 0 1 0 1 0 0 0 0 0 0 1 1 1 1 1 0 0 1 0
0 1 0 0 0 0 0 1 0 0 1 1 0 0 1 1 0 0 0 0 0 0 0 1 0 1 0 1 1 1 0 0
0 1 1 1 1 1 1 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 1 1 0 0 1 0 1 1 1
0 1 0 1 0 1 0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0 0 1 0 0 0 1 1 0 1 0
0 1 1 0 0 1 1 1 1 1 0 1 0 1 1 1 1 1 1 1 1 1 1 0 0 0 0 1 0 0 1 1
0 1 0 0 0 1 0 1 0 1 1 0 0 1 0 1 0 1 0 1 0 1 0 0 0 0 0 1 1 1 0 1
0 1 1 1 1 0 0 1 1 0 1 1 1 0 0 1 1 0 0 1 1 0 0 0 0 0 0 1 0 1 1 0
0 1 0 1 0 0 0 1 0 0 1 0 1 1 1 0 1 1 1 0 1 1 1 1 1 1 1 0 0 1 0 0
0 1 1 0 0 0 0 1 1 1 0 0 1 0 1 1 0 1 0 0 1 0 1 0 1 0 1 1 1 0 0 0
0 1 0 0 0 0 0 1 0 1 1 1 0 0 1 0 0 1 1 1 0 0 1 1 0 0 1 0 1 1 1 1
0 1 1 1 1 1 1 0 0 1 0 1 1 1 0 0 0 1 0 1 1 1 0 1 1 1 0 0 1 0 1 0
0 1 0 1 0 1 0 0 0 1 1 0 1 0 0 0 0 1 1 0 1 0 0 1 0 1 1 1 0 0 1 1
```

Cogniton ↓



S →

Fig. 2 Pariton P of "This" in bits (left), in superpixels (right)

Powers and symmetries

Using **BMP**, one can construct the successive binary matrix powers of **G** and **GG**:

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1 1	0 1	1 0	1 1	0 1	1 0	1 1	0 1	1 0
1 0	1 1	0 1	1 0	1 1	0 1	1 0	1 1	0 1
G	**Gd**	**U**	**G**	**Gd**	**U**	**G**	**Gd**	**U**

1	*2*	*3*	*4*
1 1 1 1	0 0 0 1	1 0 0 0	1 1 1 1
1 0 1 0	0 0 1 1	0 1 0 0	1 0 1 0
1 1 0 0	0 1 0 1	0 0 1 0	1 1 0 0
1 0 0 0	1 1 1 1	0 0 0 1	1 0 0 0
GG	**GGd**	**U**	**GG**

Again, a proof by recurrence will show that for any such auto-similar matrix, its square is its ⌽⊖image **Gd**, and its cube is **U**. So, at any scale, genitons and hypergenitons may be considered as automatic 3-fold symmetry generators. Then, **Gd** is a square root of **G**, since **Gd BMP Gd** produces **G**.

Also, at any scale, the binary matrix inverse of **G** is its ⌽⊖ mirror image **Gd**, the inverse of **GG** is **GGd** i.e. ⌽⊖**GG** , etc...

Here are the first 4 powers of **GGd**, **GGv** and **GGh**:

GGd	**GG**	**U**	**GGd**
0 0 0 1	1 1 1 1	1 0 0 0	0 0 0 1
0 0 1 1	1 0 1 0	0 1 0 0	0 0 1 1
0 1 0 1	1 1 0 0	0 0 1 0	0 1 0 1
1 1 1 1	1 0 0 0	0 0 0 1	1 1 1 1

GGv	**U**	**GGv**	**U**
1 1 1 1	1 0 0 0	1 1 1 1	1 0 0 0
0 1 0 1	0 1 0 0	0 1 0 1	0 1 0 0
0 0 1 1	0 0 1 0	0 0 1 1	0 0 1 0
0 0 0 1	0 0 0 1	0 0 0 1	0 0 0 1

GGh	**U**	**GGh**	**U**
1 0 0 0	1 0 0 0	1 0 0 0	1 0 0 0
1 1 0 0	0 1 0 0	1 1 0 0	0 1 0 0
1 0 1 0	0 0 1 0	1 0 1 0	0 0 1 0
1 1 1 1	0 0 0 1	1 1 1 1	0 0 0 1

This display shows that **GGd** the binary matrix inverse of **GG**, is also **GG**'s square root and also a cubic root of **U**. Then, **GGd** also has the properties of a 3-fold symmetry operator.

GGv and **GGh**, on the contrary, are their own matrix inverses and are thus 2-fold symmetry operators. Combinations of 2-fold operators with 3-fold ones produce cubic and hexagonal symmetries.

These marvelous properties, which are true at any scale, prove that a topologic universe, built from the elementary bit and *parity integration* **only**, with no other postulatum, is already able to organize itself *spontaneously* with a cubic symmetry it has all the matrix operators which will allow to build <u>three</u> equivalent (metric) axes and then grow, as described for crystals in the Int. Tables for Crystallography [HA], forming *face-centered-cubic* or *compact hexagonal* close packings.

Combined with the previously-described property, that the elementary couple 1 0 or X Y or (XX-XY) is also the elementary fractal generator, this proof shows that the "simplified"(?!?) topologic universe may be fractal at the same time as perfectly regular, and might lead to reconsider the definition of *fractality* in general.

Could these properties of the topologic universe help cosmologists? See Annex IV.

Signal transforms

A sequence may code a word in the atomic vector of a PC, e.g. **"This"**, a word with 4 characters, corresponds to the following 32-bit sequence **S**, its pariton **P** being on **Fig. 2** :

T | h | i | s
01010100 01101000 01101001 01110011

The last row is **S**, found again as the 32th integral.

In **bold** also appear the <u>second diagonal</u> **H** to be read from left to right and from bottom to top, as well as the <u>last column</u> **C**, to be read from top to bottom. Both these binary vectors have special properties:

C is the vector of the 32 successive *scalar* integrals of **S** (≠/ applied to each preceding row).

Because of the periodicity, **P** can be viewed as a *cylindric* data structure of which all integrals are generating lines. **C** becomes a 32-item topologic polygon. **H** becomes a *helix* of the cylinder; if one rolls up 64 rows, i.e. if twice the integrals are considered to form a double-size cylinder, there are two identical *double helices* running around the double cylinder.

H and **C** are transforms of **S**; and if one shifts **P**, using **N⊖P** with <u>any</u> integer value for **N**, the property will still be true for the "parallel" helix, the **N**-shifted **C** and the corresponding integral row.

The transformation **S → H** produces the <u>Helix Transform</u> of **S**.

The transformation **S → C** produces the <u>Cognitive Transform</u> of **S**.

Both transformations are <u>involutive</u>, as demonstrated by recurrence [LA4]; it means that the Helix Transform of **H** is **S**, and that the Cognitive Transform of **C** is also **S**. Each transforming matrix is its own binary matrix inverse : precisely, the mirror images of the conforming geniton **G** are the necessary transformers, according to the following basic *APL* identities, which express theorems :

```
C ≡ S  ≠.∧ Gh ◇ H ≡ S  ≠.∧ Gv
S  ≡ C ≠.∧ Gh ◇ S  ≡ H ≠.∧ Gv
H ≡  ⌽S ≠.∧ G ◇ H ≡ ⌽G ≠.∧ S ◇ S ≡ ⌽H ≠.∧ G
S  ≡ ⌽G ≠.∧ H ◇ C ≡ ⌽G ≠.∧ ⌽H    etc...
```

The transformations have properties which are close to the ones of a discrete classical *Fourier* signal transformation (more precisely, such transformations are the discrete cousins - and much more efficient since no floating-point arithmetics is ever required - of the already-famous although newly-discovered *wavelet*-transformation, known to be bound to fractal geometry: the wavelet transform also operates in a logarithmic space while the Fourier one clings to a linear space). In particular, the **S → C** one may be viewed as a space-time logarithmic transform (if one considers the bits of any sequence as the base-2-representation of integers). The main difference is that **S**, **H** and **C** form a <u>trilogy</u> for any bit sequence.

Many theorems also result from the fact that the left half of any pariton has a periodicity which is half the one of the full pariton, this property being recursive. As an example, if one replaces in a pariton, every **1** by the 2-geniton **G** and every **0** by the 2-**Z** matrix, the resulting matrix, which has a double number of rows and columns, is still a pariton, i.e. again, *every row is the binary integral of the preceding one*.

Such a process is much analogous to a *genetic differenciation*, if ones considers the successive columns of a pariton as the genes of the topologic universe : information evolves from *all-alike* in the leftmost column white vector (all 0), then a black vector (all 1), then through a grey vector (0 1 or 1 0 repeated), to the differentiated *cyclic memory* **C**. The analogy with biological and zoological facts (DNA, genes or imaginal disks) becomes striking. Some recently-observed polymer structures are also "discotic" [VM].

And all the paritons may be considered as "semi-holograms" : knowing any column or any path (theory of labyrinths) from the left towards any position in any column, allows to rebuild the whole information on the left of this column and including it. See e.g. more details on percolation and fractals in [BU] pp. 55-146. In the present case, a "lump of a broken pariton" may contain enough information so that a larger structure may be rebuilt, even from only ONE thin "information thread": the integrating process may itself be able to regenerate some lost parts!

We recently discovered that "Quadtrees", probably the most efficient parallel method in image processing for recursive block decomposition, [DA quoting [SAM]] are an application, found completely independently, of the self-similarity and growth theorems of genitons and paritons, now theoretically established as a natural Sierpiñskian fractal evolution through binary integration in *APL*.

Does our brain follow the same approach for generalised sensorial recognition? Our conjecture is YES, *first* because it will be difficult to imagine any more simple and natural model, *second* because we have successfully tried to apply these theorems to images and music, and *third* because all quantitative estimations - see e.g. Resnikoff about vision [RES] - comparing physiological data and the best known conventional methods such as the Fourier/wavelet transforms which are based on continuous functions, produce a factor measuring the "efficiency ratio: (eye-brain) towards computer" which, alas, still lies above 100,000 in favour of the brain.

The behaviour of ≠\ is (not-surprisingly) also analogous to the process of conduction enhanced by electro-magnetic waves in adjacent metallic balls, first detected by Edouard Branly and described in many papers sent to the Académie des Sciences of Paris during the last decade of the XIXth century, and, until now, not correctly explained by the laws of physics, as demonstrated at the exhibit of the Société Française de Physique at its last meeting (Caen, Sept. 1991). The percolation process in the Branly effect is exactly the same as the one induced by the first domino (or lump of sugar) falling on the next one, which falls on the following one, etc... as described in [LA4]: If the stable (or becoming-stable) state of a dynamical system corresponds to 1 and the unstable state to 0, then the initial state vector is N↑1 (N=number of dominos), when all dominos are up and the 1st one is falling down; in the end, the final state is indeed ≠\N↑1 i.e. N⍴1 or N/1 with all dominos down, i.e. the parity integral of the (spin) initial vector. Both states are respectively the last and the first rows of the genitons, i.e. of the Sierpiński triangle; moreover, the sex parity in our chromosomes still follows the same rule : X X is ≠\ applied to X Y which is itself ≠\ applied to X X : just index **'XY'** by the 2-geniton, with ⎕IO=0. Could this strong analogy be also purely fortuitous? (There are many analogies that might moreover take into account the structures of hybrids! See Annex III and [LA6]).

A pariton is indeed a dynamic automatic information-&-spin processor; it is at the same time the model of a *semi-holographic self-organising memory* (an auto-associative mini-brain of neurobits). Most physiologists now agree to think that human memory is indeed of holographic nature, without having any precise model for it.

Mutations on the transformer

An inventory has been exhaustively performed on 4-by-4 binary matrices [DU]. They are 2⋆16 i.e. 65536 possible such matrices, from **Z** (empty) to anti-**Z** (full).

Producing 5-fold symmetries cannot be obtained by 2-by-2 or 3-by-3 binary matrices, but only by 4-by-4 ones. When one looks at the (over 5000) matrices which are seventh roots of **U**, at the (#1300) ones which are fifth roots of **U**, one notices that it is impossible to derive any 5-fold matrix by ONE mutation, i.e. changing a single bit of **GG**. But such a unique mutation can produce a 7-fold transformer. Heptagons do not tile the plane and no regular heptagonal polyhedron exist so as to fill space. A *second mutation* is necessary to produce a matrix with 5-fold symmetry. Then, pentagons do not tile the plane either, but they can form 3-D structures, pentagonal dodecahedra, which are indeed observed, partly or completely (quasi-crystals, egg-shell superstructure, viruses or the newly-discovered football-shaped allotropic variety of Carbon).

Here is an example : If a first mutation occurs on the second 0 of the main diagonal of **GG**, giving **GG7**, the successive powers are:

Power:

1	*2*	*3*	*4*	*5*
1 1 1 1	0 1 0 1	0 1 1 0	0 0 1 0	1 1 0 0
1 1 1 0	1 1 0 1	1 0 0 1	0 1 1 1	1 0 1 0
1 1 0 0	0 0 0 1	1 0 0 0	1 1 1 1	0 1 0 1
1 0 0 0	1 1 1 1	0 1 0 1	0 1 1 0	0 0 1 0
GG7 ↑				

Power:

6	*7*
0 0 0 1	1 0 0 0
0 0 1 1	0 1 0 0
0 1 1 0	0 0 1 0
1 1 0 0	0 0 0 1
	U

If a second mutation now occurs on the last 0 of the main diagonal of **GG7**, giving **GG5**, the successive powers are:

Power:

1	*2*	*3*	*4*	*5*
1 1 1 1	0 1 0 0	1 1 1 0	1 1 0 1	1 0 0 0
1 1 1 0	1 1 0 1	1 0 0 0	1 1 1 1	0 1 0 0
1 1 0 0	0 0 0 1	1 0 0 1	0 1 1 0	0 0 1 0
1 0 0 1	0 1 1 0	0 0 1 0	1 1 0 0	0 0 0 1
GG5 ↑				**U**

If one mutates back to 0 the second bit of the main diagonal of **GG5**, one obtains the following successive powers of matrix **M**:

Power:

1	*2*	*3*	*4*	*5*
1 1 1 1	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0
1 0 1 0	0 0 1 1	0 1 0 1	0 0 1 1	0 1 0 1
1 1 0 0	0 1 0 1	0 0 1 1	0 1 0 1	0 0 1 1
1 0 0 1	0 1 1 0	0 1 1 0	0 1 1 0	0 1 1 0
M	**A**	**B**	**A**	**B** ...

It happens that **M** squared produces **A**, the **BMP** of which with **M** gives **B**; then, one gets the sequence **A B** which reproduces itself up to infinity... Neither **A** or **B** contain 1 in the first row or column. The 4-matrix has degenerated into a 3-matrix. The mutation was *carcinogenetic*: **M** (by the way, a Hadamard matrix), is called a "garden-of-Eden" in the Theory of Cellular Automata [WO], i.e. a state which cannot be reproduced using the generating rule (even reversed). [**G** is also a Hadamard matrix, but, fortunately, a *good* one.]

Note: **B** is also anti-**M** (not-recancellable parity inversion ?).

Pseudo-waves in topologic phenomena

"*A gravity wave is analogous to a water wave, a ripple in the curvature of space*", after [BL].

Then, what the successive iterations of ≠\ produce looks exactly like continous waves if one animates a model on a computer screen (thanks to the illusion of motion brought by the persistency phenomenon of the retina) with the following conventions:

1. Any sequence **S** is the mirror of **V**, the binary vector representation of some integer (see the first definition). The successive positions in S from left to right (or in **V** from right to left) contain the successive 0 or 1 coefficients of the power of 2 which is the current index in 0-origin during parity integration. In *APL*, even on a microcomputer, one can take care of such numbers (e.g. with 1024 bits or more, well above the IEEE integer or floating-point usual limits : a pariton with 1 Megabit has a 125-kilobyte size only, when 0 and 1 are coded in bits; current *APL* implementations on personal or micro-computers now provide a workspace area with several Megabytes: The cognitive transform of a sequence of 4 Megabits, corresponding to a pariton of 16 Terabits (i.e. *16 thousand billion bits*) has been obtained recently on a Compaq386, using *APL*⋆PLUS⋆II and 2.5 Megabytes in the clear WS with a 1-line function respecting the ISO-*APL* unextended standard...[IS]).

2. The successive rows of a mirrored pariton (thus **2⊥⍉⌽P**) correspond to the successive observable amplitudes of a periodical topologic phenomenon at a given scale. A comparison with the complexity of the *propagators* in modern physics (see e.g. the Klein-Gordon equations exposed in [FR]) is worthwhile.

The progression of ≠\ loads an accumulator (this is exactly what happens with a battery or a condenser in which discrete quanta are electrons, with a water reservoir - a dam or the ocean itself - in which discrete quanta are water molecules or droplets - depending on the scale at which quantification is considered, or, in *APL*, when one uses +\ on a sequence of integers), except that:

a) with these conventions, all topologic phenomena evolve - without any arithmetic computing - in an automatic exponential mode. The exponential (or, seen the other way, logarithmic) behaviour of phenomena - as an average - will now become the consequence of a unique underlying until-now well-hidden mechanism which produces pseudo-waves at so small a scale (in fact at all scales because of self-similarity), that the illusion of continuity may be almost perfect. See also Annex II.

b) all orders of integration are taken into account by the model, exactly as if one was able to solve a differential system with *c*-1 successive derivatives (or integrals), propagating NO more error by precision truncation, since all the data remain in bits until the final conversion to a sequence of integers or rationals : in this case, only the last bits (10 columns on the right of a pariton) need be encoded for this unique numeric output conversion, if the goal is to use a video screen which offers no more resolution than about 1,000 by 1,000 pixels in general).

But genitons themselves are paritons, and, moreover, autosimilar paritons. Then, the "curves" which may be plotted - see [LA1], smoothing the integer values obtained with genitons, are completely autosimilar : no difference can be seen on a video screen when one increases *c* beyond some threshold.

In addition, the first derivative and the first integral curves are ipso facto the same curve, with such a small ÷*c* or -÷*c* difference in phase when *c* is large enough, that one does not even remark it on the screen : thus, such perfectly irregular curves *seem* to have the same property as the exponential curve which is known to be its own integral and derivative. Idiom ≠\ *iterated* is, ab origine, able to describe dynamic phenomena; refer to the *paroxystic series* in Annex II.

The chaotic behaviour

In all paritons, the first non-0 left column contains only 1's. Then, the correlation between any 1 and any other 1 is at its maximum for all pairs of 1's, so that the correlation of this column is always 1 (correlations may vary, in statistics, within the interval {0,1}).

Let us suppose that **S** is a random sequence : (in fact, it is pseudo-random since **S** is finite). By definition, the correlation is 0 for randomness. Its cognitive transform **C** must have the same correlation because the cognitive transformation is, mathematically, an involution. (With the concept of entropy in Shannon's theory, **C** would have the same entropy as **S** by definition). No sequence other than **S** can produce **C** as a cognitive transform and the cognitive transform of no sequence other than **C** can produce **S** back. So, if the first non-0 column of a pariton has always correlation 1, and if the last column on the right has correlation 0, correlation must decrease stepwise, from column to column, from left to right. And, as an average, the pariton will appear as <u>semi-correlated</u>.

This proof would not have much importance if the expression "semi-correlated (or half-correlated) signal" was not a synonym, in signal theory, of "1/f-noise", and, as seen in many recent books of Physics, of the more fashionable idiom : "chaotic behaviour" (also, in Astronomy, of "multi-body problem", e.g. the Moon's orbit). Here are short quotations by Richard Voss (IBM Research [VO]):

"*There are no simple mathematical models that produce 1/f-noise other than the tautological assumption of a specific distribution of time constants.*"

"*Although the origin of 1/f-noise remains a mystery after 60 years of investigation, it represents the most common type of noise found in nature.*"

"*Little is also known about the physical origins of 1/f, but it is found in many physical systems : in almost all electronic components from simple carbon resistors to vacuum tubes and all semi-conducting devices; in all time standards from the most accurate atomic clocks and quartz oscillators to the ancient hourglass; in ocean flow and the changes in yearly flood levels of the river Nile as recorded by the ancient Egyptians; in the small voltages measurable across nerve membranes due to sodium and potassium flows; and even in the flow of automobiles on an expressway. 1/f-noise is also found in music.*"

One should add that 1/f-noise is found in speech (word length, tonic accent), poetry, stock exchange rates, forest-fire propagation, human demography, city growth, solar magnetism, volcano bursts, lasers, plasmas, electrocardiograms and electro-encephalograms, then probably in *APL* programs since *APL* is considered as a *tool of thought*.

So, the proposal for <u>≠\ iterated</u> (alias *periodic parity integration*), as the mathematical model for 1/f and its synonyms, may be considered as an homage to the people who introduced this powerful idiom into *APL* (the first implementation with <u>scan</u> was IBM's *APL*-SV), and to the ones who use *APL* either as a tool for research and education or as an efficient industrial-product maker.

Until now, this theory has allowed to discover, thanks to some other features of *APL* such as ⍋ and powerful related idioms, many ways of improving algorithms (which is hardly feasible with other programming languages).

The pariton model also fits observations of Biology, from G. Mendel's genetics to the imaginal disks of larvae, e.g. the 4+8 disk structure of Drosophila [SUZ] (see also [LA1, LA2] and the expression in Appendix III, producing a replicative automaton, a new life-game), of Chemistry (from Mendeleiev's periodic table [LA1] to Kopelman's Sierpiñskian model for "fractal" heterogeneous catalysis [KO] and to JP. de Gennes' chain-polymer models/automata [DG]), of Crystallography (cfr. [HAH] and its ref.), etc... Blood groups and the Rhesus factors are probably also correlated. The model may have some interest in Linguistics [LA2]. Genitons are connected to recursive algorithms [STE], combinatory processes (Pascal's binomial coefficients) [LA3], finite-difference models (see e.g. [LO] for light absorption and transmission by clouds), etc...

Given the length constraint for this paper, it is impossible to give more details. However, it seems interesting to show the important connection with Fibonaccian processes, at least on one last example other than the simple mention that ≠\ is <u>per se</u> a generalised binary Fibonacci auto-propagator (each parity resulting from the preceding ones, i.e. given by the (+ modulo 2)-function i.e. ≠.

Properties of G as an integer matrix

If **G** is now considered as an *integer* matrix, iterated +.× matrix products return the successive matrix powers of **G**:

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
1 1	2 1	3 2	5 3	8 5	13 8	21 13	34 21
1 0	1 1	2 1	3 2	5 3	8 5	13 8	21 13

Due to the properties of the matrix product, all the items of these matrices belong to the Fibonnacci series; moreover, every matrix is symmetric and contains the two successive Fibonacci numbers in its rows (or columns). In addition, the matrix sequence is itself a Fibonacci sequence, each matrix being the arithmetic sum of the 2 preceding ones. The parities of this matrix sequence (applying **2|**) reproduce, of course, the successive power sequence of **G** when the **BMP** is applied. Matrices with a power multiple of 3, have **U** as their parity matrix. It is interesting to mention that the Fibonacci series is also connected to 5-fold symmetry and to the *golden section* [KE]; successive Fibonacci numbers also play a role in the spirals of vegetal growth (see [JE] about sunflowers and pine-cones). Jean has also detected that the 2-geniton was the *growth matrix* for tree-structures.

Other biological connection

About cyclic processes, Winfree proposes [WIN] to create a rotor "as the self-maintaining source of a rotating wave" to explain some rather frequent biological behaviour (e.g. in mushroom mycelium or for the regeneration of amputated limbs by some animals) : two rectangles are put into contact; in one of the rectangles, a concentration "wave" oscillates between two states; in the second one, there is no wave. Such a sketch may be exactly described by the two vertical halves (as well as by the two horizontal halves) of the 2-geniton :

```
1 | 1
1 | 0
```

no "wave" | oscillation

and extending the sketch to larger autosimilar genitons, or even to paritons, which will always exhibit in their right half a more diversified behaviour than in their left one : in any square pariton, the left upper quarter is always the same as the left lower one, since the period of the left half is half the period of the right half. The property is still more general than in Winfree's model, since it stands at all scales, for this upper left half is itself a pariton; so, the property is recursive.

About the initial postulatum

The assumption viz. *the Main Mechanism of the topologic universe is parity integration*, is, in fact, **NOT** a postulatum. It was found at the end of a strictly experimental démarche, as the *nucleus* of information processing, after several years of *algorithmic compression*, with the help of *APL*. Algorithms, i.e. *modi operandi* for data handling in some fields of knowledge (image, graph, text and natural-language processing, crystallography, chemistry, applied mathematics, inter alia), were submitted to a strong compression in parallel: it consisted in trying to simplify and/or to discover some programming constructs or just tricks, and, fortunately, new theoretical concepts, which could lead, in each case, to more general, faster and shorter codes than the previous versions, which were thrown away. Such a process must have an end for every algorithm for a particular problem, the limit being the nucleus of this particular problem. The surprise was great when the answer came out that, for ALL tested algorithms, there was ONE common nucleus, which was identified as parity (or Boolean) integration, then ≠\ in *APL*. (In no other programming language, such a construct is intuitive except, perhaps, in C++ for parallel machines : the SCAN operator is also provided, with another definition for the first item; but scans do not even exist in the present proposal for ISO-FORTRAN90).

Algorithmic compression has another advantage: WS and disks can contain more functions; see [LA5] (over 2,000 functions in one WS on a DD-diskette).

Then a question about the History of Sciences arises: Sir Isaac Newton (1642-1727) invented the Theory of Fluxions, i.e. calculus, i.e. differenciation and integration, in order to explain Kepler's laws as well as his own observations on gravitation. Developments came from Leibniz (1646-1716), Laplace (1749-1827), Lagrange (1736-1813), Cauchy (1789-1857), Hamilton (1805-1865) and many others such as Maxwell (1831-1879). But all these great scientists (except Leibniz who also studied binary algebra: he knew some Chinese binary arithmetics) could not or did not take enough advantage of Boole (1815-1864)'s work. What would Newton have invented for integration if he had considered logics and binary algebra also? He reasoned in the universe of continuous functions or numbers: there was nothing else by that time, not even any micro-computer with efficient *APL* implementations. No discrete phenomenon was yet taken into account as some scientists' everyday job. Einstein, Planck, Rutherford, Schrödinger, Pauli, Dirac and de Broglie came much later. Quantum Mechanics is 60 years old, *APL* only 26 (a youngster!).

Must **The General Laws of Physics**, i.e. of Nature (and not only etymologically: "*Physis*" meaning "*Nature*" in Greek), if they are supposed to converge within a **T.O.E.** ("Theory Of Everything" or a **G.U.T.** ("Grand Unification Theory"), come from Particle Physics or Astrophysics only? Should they be different in Chemistry, in Biology then in

Genetics and Linguistics, in Crystallography, in Information Processing then in the field of Programming Languages? May *The Solution* also emerge from Computer Science, and, particularly, from the richest construct ever built in this branch of knowledge, which is *APL*, thanks to Iverson's genial intuitions?

As a first conclusion, let us quote Prof. Steven Weinberg, from a lecture given in Cambridge, G.B. to Paul Dirac's memory, [WE]:

"My colleague from the University of Texas, John A. Wheeler, thinks that, when, at last, we know the ultimate laws of Physics, we shall be most astonished of the fact that they had not previously appeared as obvious".

Paul Dirac himself also said about theories [SAL] :

"The ultimate goal is to obtain suitable starting equations from which the whole of atomic physics can be deduced. We are still far from it... ...However, the present quantum electrodynamics does not conform to the high standard of mathematical beauty that one would expect for a fundamental physical theory, and leads one to suspect that a drastic alteration of basic ideas is still needed".

Many *APL* users already appreciate the power of ≠\ in various fields; see e.g. the clever use of it for fast graphic clipping in [SP] (so clever that the *APL* model was adopted by IBM for a windowing system on some graphic stations).

But the ≠\ construct is much more than a simple *APL* idiom: it expresses the main interaction at the bottom-most level, the one of the fundamental parities. A recent discussion with one of the most famous mathematicians confirms the point [TH] : René Thom thinks that the "elementary mechanism" is a propagation.

Indeed, the topologic universe, based only on ≠\ i.e. parity propagation, alias a dynamic extension of Wolfgang Pauli's Exclusion Principle (which is the basis of Quantum Mechanics) lies still far from our complex Universe; but it is closer to it than most sophisticated models presented individually in many fields of Science, and seems to fit automatically, i.e. with no other effort, all the domains we explore systematically.

The "suitable starting equations" will hardly be found: real-Universe understanding would imply very-high-order differential equations, leading to formulae with many non-linear terms (5 terms are used in Molecular Dynamics and for Solitons; 7 terms appear in the present-time formula which describes the Moon's orbit, trying to take into account some chaotic behaviour that was unknown in Newton's and Laplace's time).

Geniton and Paritons are equivalent to computing automata which consider all possible orders of integration or derivation a priori, even billions or more... with intrinsic self-similarity. Moreover, the results of parity integration are always perfectly accurate at all stages and scales. And this new life-game does not use any extra ad-hoc rules. Most signals observed in Nature, see [VO], are *half-correlated*. Models based on genitons and paritons exhibit naturally, without having been ever forged on purpose to do so, the so-called mysterious (cfr. [VO]) behaviour, common to such signals.

Furthermore, the geniton-model has shown that symmetry operations can replace even binary computing in many places (would this property not even be more general?). And no computer on this planet is indeed able to perform, at the elementary level (one bit) within any couple of its active registers, anything else than Boolean operations on *couples* of bits, which can only be **0 0** , **0 1** , **1 0** or **1 1**. Indeed, all other Boolean primitives can be deduced from either ⍱ or ⍲ ([LA2] identities p. 731), in a *static* way. The *dynamic couple* ≠\ and MAJ (∧ is a special case of the MAJ function as shown in Appendix I) controls the evolution of the dynamic universe and allows to build the whole of information processing also with some *fuzziness* (see [LA2], p. 712).

Many Boolean automata which have been proposed so far use asymmetric functions (∧ or ∨) in a symmetric way, see [KAS], [WO]. This is also true for + in Conway's life game. A very recent reference [STA] explains Wolfram's, Kauffman's, Ising's and Herrman's models in which ≠ indeed plays the main part, still combined with ∧ in a symmetric way. Conversely, the parity-integration model is based on the UNIQUE symmetric function ≠ used in a completely *asymmetric* way (due to the effect of the **scan**). This subtle change in the way of conceiving a model of topologic universe makes indeed all the difference, introducing ab initio an oriented arrow for *time*. Then, the irreversibility of time need not be postulated anymore : scan, i.e. Propagation, indeed creates "symmetry breaking".

Small *APL* identities (theorems) show that the genitons correspond to the "balanced-state" of the system: Given **P**, the pariton of any informational sequence **S**, then, **Pn**, the pariton of the "negative" of this sequence **~S**, or, rather, **1≠S** , is **P≠1⊖G** where **G** is the conforming geniton. And the cognitons of **S** and **~S** only differ by one "spin", the last item. In bits, any information coming from numbers, texts, signals, genes or concepts, becomes integrable *ad infinitum*.

Propagation of information along an axis **S** of the topologic universe implies that two orthogonal fields (**H** and **C** which could be named *topoelectric* and *topomagnetic* in a *trihedral model*) are coupled to it and between themselves, (as shown by Maxwell). And the path of an electron within a magnetic field is indeed *helical*, just like it occurs in the Helix transform of the cylinder model.

Acknowledgments

The author's thanks go to over 500 authors and editors of recent books (a very small list is given in the references) as well as to the people who manage various libraries in Saclay. Some friends have also lent or given many books, reprints, papers and computers without which it would have been impossible to verify the consistency of such a storming puzzle; other ones have listened, checked, discussed, spending much time on the topic, and even sometimes learning *APL* just in order to understand this strange notation in which theorems are programs at the same time. Some precious help has been provided by meeting organisers: F. Collot in Biomathematics, M. Locquin in Botanics and Transdisciplinarity, with support of U.N.E.S.C.O., group GESYS (Paris), J. de Kerf (Belgium) in *APL* & Mathematics, The Academy of Sciences, Moscow, Section of Cybernetics & A.V. Kondrashev, as well as SOV*APL* (Russian *APL*-Association), organisers of the Russian-French Obnisk *APL* Seminar in Feb. 1992. Many thanks also to French mathematicians M. Dumontier, A. Mary, and, especially, C. Cortet (CEA) who did transform some conjectures into theorems, using "conventional" mathematical notation, so that what is simple with *APL* can also now be grasped by "normal" people. Intellectual sustenance has also been brought by Dr J. Hainaut (medicin), D. Gris (zoology & genetics), C. Chachaty (NMR & RPE spectroscopy), E. Soulié (optimisation), R. Conte (physics & mathematics), J.J. Girardot (*APL*), as well as J. Langlet - the author's wife (Theoretical Biochemistry) who has been suffering from her husband's infection by the incurable *APL* virus for almost 20 years.

≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠

Annex I: Primitive Operations

The "primitive" arithmetic operation + on non-negative integers is, in fact, not a true primitive for the computer : it can be described, using 3 elementary bit-registers : first, registers A & B contain the lowest-bit of the operands; register C is used for the carry and contains 0.

Array **R** in 0-origin is 2 2 2⊤ι8 and the integral ≠⌿**R** returns the parity of each column of **R** :

```
0 0 0 0 1 1 1 1  C
0 0 1 1 0 0 1 1  A
0 1 0 1 0 1 0 1  B

   ≠⌿R
0 1 1 0 1 0 0 1
```

Every column represents all the possible configurations of the 3 registers without and with carry (≠⌿ is used in order to show the result in the same way as addition is usually performed).

Remark. A noticeable identity is also the fact that scalar integration of the columns of **R** which contain binary items does not depend either on the order of the items or on the ≠ primitive itself : =⌿**R** returns **0 1 1 0 1 0 0 1** i.e. the same result as ≠⌿**R** does.

This is the result of adding A B C modulo 2, while the *carry* which has to be *left-propagated* into C for next-bit processing is given by the *majority* of the columns of **R** which is:

0 0 0 1 0 1 1 1 for all the respective 8 cases.

This defines completely the *arithmetic sum* and, hence, all consequent arithmetic constructs.

And the **BMP** is the composition of ≠ and ∧. Then, ∧ may also be viewed as the *majority* of 3 registers of which one will always contain 0. After the *majority operation,* a *reduction by* ≠ i.e., another time, a scalar *parity integration*, is performed. It has also been shown that, in the case of matrices of the geniton family, matrix inverse and matrix powers may be replaced by mirror symmetries or rotations which produce the same results.

So, 2 elementary bricks, called *binary integration* and *majority*, allow to rebuild the main operations of data processing. Growth has already been dealt with, using ⊂ and ∧ (⊂ is the third brick, but Iverson's monadic < could be used as well: no conflict can occur, because the *enclosed* or *boxed* arrays are never scalars). From *binary integration*, one rediscovers the properties of finite differences in any discrete algebra, then, with infinitesimal intervals, one may study continuous functions. And the *majority* brick is, evidently, the main brick of the Theory of Decision also, see [LA2].

Annex II. The paroxystic series

The principle that, *in a finite periodic dynamic topologic system, the successive states correspond to the successive integral sequences* leads to the following idea : In a pariton, each row can be seen as the binary representation of a (large) integer, the highest bit being on the right.

In a condenser or a battery, or, simply, in the sea, the non-stop accumulation by "wave propagation" starts with one electron, one droplet and ends with a spark (the condenser is fired) or a surf on the shore. Human life follows the same scheme, from the tiny quantum, a fecondated ovum with successive different phases of growth, till the last - discontinuous - moment of death (the most discontinuous efficient modulo-tool being the guillotine). Empires, doctrines, theories as well as stars also follow this scheme. Brusque isolated events which occur more or less periodically, such as earthquakes, avalanches or epidemies, (now indeed recognised as fractal and chaotic), in general measured with a logarithmic scale, on one hand, and the propagation (scan, accumulation, integration) of parities in the pariton, coding, in a 2-base logarithmic way, a sequence of numbers within its rows, on the other hand, look similar. The genitons are the *prototypes* of paritons (the Boolean difference ≠ of the pariton of any sequence **S** and of the pariton of ~**S** is always 1⊖**G** with **G** the conforming geniton). So, using the *cylinder* model, let us recompose the numbers from the successive rows of the genitons, using 2⊥⍉Φ**G** in *APL*. Genitons are symmetric: so ⍉ may be omitted; yet, ⍉ is kept in expressions to take care of paritons. Then, let us look at the periodic discrete results, smoothing and plotting the curves so as to give them the familiar look we observe with sismographs, spectrometers or any common recording device. Such curves and their derivatives were already published in [La1] and look like earthquake signals, X-ray or NMR spectra, with small satellite peaks which are indeed observed (e.g. in the secondary earth tremors).

In a medium-size binary matrix, e.g. a 256- or 512-geniton, the influence of the lowest (left) bits is negligible; anyhow, only about 50 rightmost bits can be considered in IEEE precision. Then, studying a large scale of genitons with different sizes, it is possible to formulate the general law which gives, directly in integers, the succession of numbers which mimic so strongly the sampling of natural phenomena at successive regular intervals; this law corresponds to the *paroxystic series* :

1 is the first item of R. Iterating R←R,R×2+¯1↑R 5 times produces the 32 first items i.e. the sampling of the "average" reversed periodic signal with period 32 i.e. the result of Φ2⊥⍉Φ**G** if **G** is a 32-geniton :

1 3 5 15 17 51 85 255 257 771 1285 3855 4369 13107 21845 65535 65537 196611 327685 983055 1114129 3342387 5570645 16711935 16843009 50529027 84215045 252645135 286331153 858993459 1431655765 4294967295

As signalled by C. Cortet, the series contains many Fermat numbers which differ by 1 from the even powers of 2 i.e. powers of 4. Smoothed curves of the *logarithms* of such series as functions of "time" will produce, if *c* (a power of 2) is the period, saw-tooth signals which, when *c* is large, cannot be distinguished from straight segments on the video screen : this suggests that when a complex phenomenon is approximated by exponential curves, so by straight segments in semi-logarithmic plots, small "anomalies" are considered just as parasites, although they might, in reality, correspond to the intrinsic behaviour (in fact unknown, then qualified "chaotic") of the phenomenon. As an average, in the curves by R.F. Voss about music [VO], all musics have the same spectral density. The 1/f behaviour is just an average. Finer analysis reveals more complex (not-understood) shapes.

More generally, paritons, when encoded this way, easily allow to mimic observed spectra, especially EPR (electronic paramagnetic resonance) ones: some specialists (physico-chemists, biologists), who did not know that the spectra resulted from a short *APL* one-liner with no arithmetics (only ≠\ iterated) except 2⊥ encoding and parabolic smoothing before plotting - in less than one second, with plot on a PC - were often able to "identify" the chemical products which could have produced the spectra!

Annex III. Genesis and the new Life Game.

The role of ≠\ as the "motor of the system" may be discovered with no axioms involving metrics or natural numbers, using the following reasoning, which, then, may not be concerned by the consequences of Gödel's theorem (Gödel had to use the theory of numbers to prove his theorem in 1931):

Given **EGO** as any entity (physical as well as psychological), the only obvious property of **EGO** is that **EGO** is *different* from what is not itself. Then, if **EGO** is a part of a dynamical system, it tries to *propagate* its unique property on its closest **neighbour** (not neighbours as in the Theory of Cellular Automata [BU, WO, STA], because, in a ≠\ not-Euclidian universe, there is always ONE closest neighbour in the fractal hierarchy of parities, see Pauli's exclusion principle, as well as the definition of ⍋ or ⍒ in *APL* for which a decision has to be taken according to another left/right sub-parity in case of equal items in the argument) which is **~EGO** , or, rather, **1≠EGO** , by definition, at the "beginning". So, ≠\ i.e. *binary integration*, IS BORN...

From this almost trivial reasoning, the rule of a new Game of Life emerges at once, e.g. for 2-dimension binary arrays, as the following algorithm: *The results of the binary integrations ≠⍀ and ≠\ (along both axes) with a respective* 1⊖ *and* 1⌽ *shift shall be* <u>*different*</u>*; the result is the new array.* The automaton will expand because of a **0,0⍪** catenation at each step, which also allows to use 1⊖ and 1⌽ without side effects. If the initial array size is e.g. 16 16, the automaton will reproduce the pattern exactly, with an additional "going-away" clone, every 16th step, forming also intermediate pseudo-hybrids between itself and its symmetrics, automatically. Every iteration which corresponds to an odd multiple of 8 (the half-period on both axes) will produce astonishing patterns. Try, with a sprite, font, icon or pixel editor, to generate some initial arrays and send the nicest patterns (or films) to the author. Thanks. ROBY the robot (provided by T. Scherer) is a good example to start with:

```
ROBY←0≠(16⍴2)⊤0 0 0 960 65 505 20239 30984
          2312 30984 20239 505 65 112 0 0
```

Then, with **B**←ROBY , just ITERATE at will, if you like, until infinity :

```
B ← (≠⍀ 1⊖ B) ≠ ≠\ 1⌽ B←0,0⍪B
```

and plot the successive images of **B** as bitmaps or use semi-graphics (very fast on PC) if possible. Split this expression into smaller ones in order not to get WS FULL if you intend to form big arrays, especially when bits are coded as integers (*APL*⋆PLUS I). Also, beware of the generated army of robots and "pseudo-hybrids"!

The internal structure of pseudo-hybrids can be studied, using the non-symmetric mirror-images of genitons, i.e. ⊖**G** or ⌽**G** . Automata in 3 or more dimensions are easily written in one line of *APL* code, also with more complex rules, involving ≠ and its scans only, of course, as THE *essential* primitives.

Annex IV. From Genitons to Instantons (Theoretical Physics)

Let us start with some quotations:

"*There is a mechanism that can concentrate the probability distribution of one quantity, the cosmological constant, precisely at zero. However, it is not clear whether the same mechanism, concentrates the probability distributions of other coupling constants, in a similar way. ...But the question of whether observed physical constants, have precise values, or a probability distribution, could be answered only by going to the* <u>*underlying fundamental theory*</u>*. I would put the chances 50:50.*"
S.W. Hawking, (1990) [HAW, p.266].

"*The first [question] is whether summing over topologies can or should be done in quantum cosmology. I Frankly don't know, but I think it is likely that it cannot be avoided.*"
L.Susskind [SUS].

"*The single parent universe propagates in a plasma of baby universes.*"
A. Strominger [STR, p. 286].

"*The instanton therefore represents nucleation of a small toroidal baby universe.*"
A. Strominger [STR, p. 325].

Then, a precise description of the hypothetical "instantons" is provided by the whole set of genitons; and the mathematical proof of it is evident: If paritons behave like *strings*, in the cylinder model, because of the periodicity of integrals, genitons behave like *toroids* : genitons have a diagonal symmetry which other paritons have not; every column in the flat matrix is also the parity integral, given by ≠⍀ , of the preceding column on the left; and the first column is always the parity integral of the last one; then, the cylinder or string can also be "stuck horizontally" in order to form a *topologic torus*. The connection with theoretical physics and recent concepts in cosmology becomes then stronger and stronger, the model being developped without equations, without summing anything (except modulo 2, by definition), without probability distributions, and with a very universal <u>underlying fundamental theory</u>, just involving *symmetry*, *parity* and *asymmetric propagation* of this latter, everything else becoming only a <u>consequence</u>, thus no more a <u>cause</u> of the phenomena.

≠\≠⍀≠\≠⍀

References and connected books

[AM] A. Ambrózy (ed.) Noise in Physical Systems, including 1/f-Noise, Biological Systems and Membranes, Akademia Kiadó, Budapest, Hungary, ISBN 963-05-5778-9, (1990).

[BE] N.V. Belov, Yu.G. Zagal'skaia, G.P. Litvinskaia & Yu.K. Egorov-Tismenko, Atlas of the space groups of the cubic system, Nauka, Moskva, USSR (1980) [in Russian].

[BL] D.G. Blair, The Detection of Gravitational Waves, Cambridge University Press, ISBN 0-521-35278-9, p. 46 (1991).

[BU] A. Bunde & S. Havlin (eds.) Fractal and Disordered Systems, Springer, Berlin, ISBN 0-387-54070-9 (1991). See also [STA].

[CH] G. Chaitin, A Computer Gallery of Mathematical Physics, (in *APL2*), IBM Research Report, Yorktown Heights, NY, USA, cfr. pp. 46 & 52 (March 23rd, 1985).

[DA] L.S. Davis, Shape Representation, in: R.A. Vaughan (ed.), Pattern Recognition and Image Processing in Physics, Proc. of the 37th Scottish Univ. Summer Schools in Physics, Dundee, 1990, ISBN 0-7503-0121-X, p. 114-117 (1991).

[DG] P.G. de Gennes, Introduction to Polymer Dynamics, Cambridge University Press, ISBN 0-521-38172-X, (1990); and: Scaling Concepts in Polymer Physics, Cornell University Press, Ithaca/London, ISBN 0-8014-1203-X (1979).

[DU] M.J. Dumontier, Genetics of the XOR-function on Boolean Matrices, accepted in *APL*-CAM Journal, Belgium (1992).

[FR] E.S. Fredkin, D.M. Gitman & Sh. M. Shvartsman, Quantum Electrodynamics with Unstable Vacuum, Springer, Berlin, ISBN 0-387-52935-7 p. 156 & sq. (1991).

[GR] J.B. Griffith, Colliding Plane Waves in General Relativity, Clarendon Press, Oxford, G.B., p. 468 & sq. (1991).

[HAH] Th. Hahn (ed.) International Tables for Crystallography, Vol. 4. "Space Group Symmetry", Reidel Publishing Company, Dordrecht/ Boston, ISBN 90-277-2280-03, fig. 2.8.1a p. 23; see also the references to M.J. Buerger's work and H. Arnold's contribution p.70 & sq. (1987).

[HAW] S.W. Hawking, Wormholes & Non-simply Connected Manifolds, in: Quantum Cosmology & Baby Universes, (S. Coleman & al., eds), World Scientific, Singapore, ISBN 981-02-0345-4, vol. 7, p. 245-266 (1991).

[HO] J. Horgan, Profile: Physicist John A. Wheeler, Questioning the "It from Bit", Scientific American, pp. 36-37, (June 1991).

[IS] ISO8485, ISO, Geneva, Switz., p. 91 (1989).

[JE] R.V. Jean, Mathematical Approach to Pattern and Form in Plant Growth (Phyllotaxy), J.Wiley & Sons, New York, ISBN 0-471-88357-3, pp. 143-144 (growth matrix), (1984).

[KAL] L.H. Kauffmann, Knots and Physics, World Scientific, Singapore, ISBN 981-02-0343-8, cf. The Lie algebra of SL(2) p. 143, Eigenstructures, the M+ and M- Matrices, p. 388, Q-spin networks... p. 459 (1991).

[KAS] S.A. Kauffman, Boolean Functions and State Cycles (caption in the paper: "Antichaos and Adaptation"), Scientific American, Vol. 265, No 2, p. 80 (Aug. 1991) [see also : J. Theor. Biol. vol. 22, 437 (1969)].

[KE] J. de Kerf, Fibonaccitis, *APL* XII Studiedag, Antwerp, Belgium (Nov. 1991), also : *APL*-CAM Journal, 13, 4, 942-977 (1991) [in Dutch].

[KO] R. Kopelman, Diffusion-controlled Reaction Kinetics, from : D. Avnir (ed.) "The Fractal Approach to Heterogeneous Chemistry", ISBN 0-471-91723-0, p. 296 (1989).

[KR] E. Krause, Taxicab Geometry, Addison Wesley Publishing Company, USA (1975).

[LA1] G.A. Langlet, Paritons & Cognitons, Towards a New Theory of Information, *APL*-CAM Journal, 13, 3, 399-432 (1991).

[LA2] G.A. Langlet, New properties of the Paritons,... (Part II), *APL*-CAM Journal, 13, 3, 709-743 (1991).

[LA3] G.A. Langlet, Variations sur Sierpiński, Pascal & Fibonacci, *APL*-CAM Journal, 13, 2, pp. 375-389 (1991) [in French].

[LA4] G.A. Langlet, La Propagation Asymétrique de la Parité..., BioMath, VOL XXIX, No 115, pp. 5-42, Paris (1992) [in French].

[LA5] G.A. Langlet & M.V. Locquin, Présentation de *Factotum*, *APL*-XII, Antwerpen, Belgium (Nov. 1991) [in French].

[LA6] G.A. Langlet, The Structures of Hybrids in the new Game of Life, with simple animated models [accepted in Biomath] (1992).

[LI] S. Lipschutz, Theory and Problems of Discrete Mathematics, McGraw-Hill Inc, New York, USA, ISBN 0-07-037981-5 (1976) & : Mathématiques discrètes, cours et problèmes, McGraw-Hill, Paris, ISBN 2-7042-1233-3 (1990).

[LO] S. Lovejoy, P. Gabriel & G.L. Austin & D. Schertzer, from : Fractal Aspects of Materials: Disordered Systems, M.R.S. extended abstracts, p. 205 (Nov. 1988).

[MA] V.G. Makhankov, Soliton Phenomenology, Kluwer, NL, ISBN 90-277-2830-5, 108 (1990).

[OT] E. Ott, Strange attractors and chaotic motions of dynamical systems, Rev. Mod. Phys., 53, 655 (1981).

[PA] J.K. Park, K. Steiglitz & W. Thurston, Soliton-like behavior in automata, Physica 19D, 423 (1986) (Filter-automata).

[PO] W. Poundstone, The Recursive Universe, Contemporary Books, Chicago, USA (1986).

[RE] H. Resnikoff, The Illusion of Reality [Theory of Vision], Springer, Berlin/New York, ISBN 0-387-96398-7 (1989).

[RO] E.E. Rosinger, Non-Linear Partial Differential Equations (An Algebraic View of Generalized Solutions), North Holland Mathematic Studies, 164, ISBN 0-444-88700-8, Elsevier, NL (1990). [see the chapter devoted to Colombeau's algebra].

[SAL] A. Salam, Unification of Fundamental Forces, Paul Dirac's Conf. (1968), "Difficulties in Quantum Electrodynamics"), Cambridge University Press, ISBN 0-521-37140-6, p. 143 (1990).

[SAM] H. Samet, "The Design and Analysis of Spatial Data Structures". "Applications of Spatial Data Structures", Addison-Wesley, GB (1989).

[SP] T.L. Springall & G. Tollet, A Shading Approach to Non-Convex Clipping, *APL* Quote Quad, Vol. 20, No 4, p. 369-372 (July 1990, *APL*90-Copenhagen).

[SUS] L. Susskind, Critique of Coleman's Theory of the Vanishing Cosmological Constant. Same as [HAW], p. 347-351.

[SUZ] D.T. Suzuki, A.J.F. Griffith, J.H. Miller & R.C. Lewontin, An Introduction to Genetic Analysis, Freeman & Co., New York/Oxford (1989) 750 p. chapter 22, Fig. 32, 34, 41 & text.

[STA] D. Stauffer, Cellular Automata, Chapter 9 of [BU], pp. 296-321 (1991).

[STE] I. Stewart, in Pour la Science, French Ed. of Scientific American, No 142, p. 103 (1989).

[STO] T. Stonier, Information and the Internal Structure of the Universe, Springer, London, G.B. ISBN 0-387-19599-8 See p. 34,83, 111-4 & e.g. Appendix A p. 121 & 131 (1990).

[STR] A. Strominger, Baby Universes, same as [HAW], pp. 269-346.

[TH] R. Thom, Private communication (1992).

[VO] R.F. Voss in: "The Science of Fractal Images" (Peitgen & Saupe, eds.), Springer, New York, ISBN 0-387-96608-0, p. 31,39-40, fig. 1.9 (1988).

[VM] I.G. Voigt-Martin, H. Durst et al. Microstructure of a Discotic Polymer as revealed by electron diffraction and High-Resolution Imaging, Angewandte Chemie, Ed. in English, 28, 3, p. 323 (March 1989).

[WE] S. Weinberg, Conference to Paul Dirac's memory, Cambridge, GB in Feynman & Weinberg: The Character of Physical Law, Cambridge University Press (1967).

[WES] G. Weisbuch. Dynamique des Systèmes Complexes, CNRS, Paris (1989) [translation into English: "Complex Dynamic Systems", Addison-Wesley, New York (1991)].

[WIL] S. Wilson (ed.) The Effects of Relativity in Atoms, Molecules, and the Solid State, Plenum Press, New York, ISBN 0-306-43888-7 (1991).

[WIN] A.T. Winfree, The Geometry of Biological Time, Springer, N.Y., ISBN 0-387-9373-7 fig. 5, p.233 & fig. 10, p.239 (1980).

[WO] S. Wolfram, Theory and Applications of Cellular Automata, World Scientific, Singapore, ISBN 9971-50-123-6, p.1 & fig. 5 p. 13 (1986).

After submission of this paper, connections with studies in advanced physics, mainly Russian ones, appeared. Here are references giving an idea of what cannot be widely described in a paper devoted to *APL* (not to symmetry or chaos or integral systems):

[DO] V.V. Dodonov & V.I. Man'ko, Integral Systems, Solid State Physics and Theory of Phase Transitions, Pt. 2, Proc. of the XVIIIth Int. Coll. on Group Theoret. Methods in Physics, Moscow, Russia, June 1990, Nova Science, New York, USA, ISBN 1-56072-038-7 (1991).

[EV] R.A. Evarestov & V.P. Smirnov, Application of Site Symmetry in the Solid State Theory, in [DO] p.81-4 (1991). Conclus. 6 *magnon* symmetry.

[KON] S.E. Konstein & M.A. Vasiliev, Higher Spin Superalgebras... in [DO], p.233-6 (1991).

[KOS] O.S. Kosmachev, On the Projection Operators of the Irreducible Representation of Symmetric Groups, in [DO] pp. 287-289 (1991).

[KUB] Y.A. Kubyshin, J.M. Mourao & I.P. Volubujev, Graphic Schemes of Root Ordering in simple Lie algebras, in [DO] pp. 261-4 (1991).

[KUR] E.V. Kurmyshev, S.A. Dirov & A.V. Naumov, On some possibilities for description of associative media by Spin-Glass models, in [DO] pp. 147 (1991).

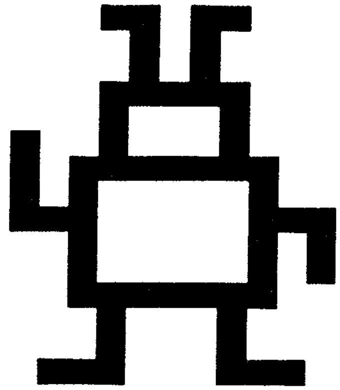
[PI] R.I. Pimenov, New Spacetimes - Anisotropic & SemiRiemannean, in [DO] pp. 257-60 (1991). "Anisotropy Degenerated" leading to Maxwell theory.

[SA] A.V. Samokhin, Symmetries and Equivalence of Differential Equations, in [DO] pp. 69-72 (1991); [leading to the Kortveg-de Vries equation].

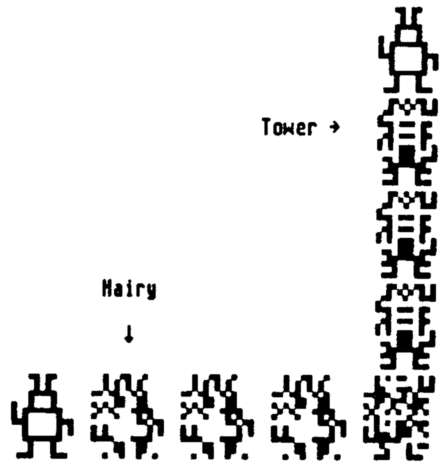
[YU] V.I. Yukalov, Group of Transformations for continuous Iteration, in [DO], pp. 297-300 (1991); [a new method using self-similar transforms].

[ZA] G.M. Zaslavsky, R.Z. Sagdeev, D.A. Usikov & A.A. Chernikov, Weak Chaos and Quasi-Regular Patterns, Cambridge Nonlinear Science 1, (Prof. Chirikov & al. ed.), Cambridge University Press GB, ISBN 0-521-373174-4 (1991). See, inter alia, the Kolmogorov-Arnold theorem (KAM theory), p. 28, the non-trivial effects of discretization, p. 53-5, the stochastic spinning of satellites, p.55-7, dynamic patterns, p.148 & 170, phyllotaxis & 5-fold symmetry, p. 224 & sq.

≠\≠\≠\≠\



ROBY after 64 iterations
becomes : twice ROBY +
once a pseudo-hybrid +
3 times the "hairy" hybrid +
3 times the "tower" hybrid



ROBY with its clones
and various pseudo-hybrids
after 96 iterations