APL Econometric Planning Language (EPLAN)

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ABSTRACT

The APL Econometric Planning Language (EPLAN) is an IBM Installed User Program providing features to assist the construction and application of econometric models in an interactive environment.

Its design takes into account the nature of the econometric modeling process, which ideally requires a man-machine dialogue. Main objectives have been: User-oriented terminology (model language), functional capabilities, generality and fast response.

The elements of the model language observe a hierarchical structure. The atomic items are the economic variables, i.e. time series or cross-sectional data, and constants. Subsequent stages comprise operations on economic variables, equations and models. This logical structure is maintained by the use of suited APL data structures.

The functional capabilities include several techniques of single equation parameter estimation, the solution of non-linear models and routines for data transformation, documentation and file handling.

Generality is ensured by defining EPLAN open and transparent. The various programs are considered building-stones for the user's specific application. He can optionally mix EPLAN programs with programs of his own or from the APL Public Library.

Fast response is automatically given by the excellent time-sharing characteristics of the APL host system.

The paper describes the design of EPLAN, with emphasis also on its relation to APL, and applications in the fields of economics and business planning.

1. Introduction

The APL Econometric Planning Language (EPLAN) was designed and implemented at the IBM Philadelphia Scientific Center to provide a medium within which a user would be able to manipulate economic variables and to construct and utilize econometric models. It has now been made available to a wide audience as an IBM product (Installed User Program #5796-PDW, see also the references) and is being used by a number of large industrial and governmental IBM customers.

In this paper we attempt to give an idea of our basic design objectives, which we believe to have some general validity and utility. Then we outline the system capabilities and characteristics. We can be relatively brief in view of the rather complete description available in the references.

However, it may now be a good time to add appendices which contain material not readily available elsewhere (in the form pertinent for EPLAN):

A1. a concise but fairly complete survey of the basic statistical terms and techniques,

A2. a tutorial for the uninitiated who wishes to demonstrate EPLAN to himself at a (real or imaginary) terminal.

While not a harmonious whole, these various materials should benefit potential users of EPLAN, or indeed other econometric packages. We hope also that the paper may stimulate the APL community to consider econometric techniques in areas where they have not yet found wide use, yet where they are clearly applicable, and to design and implement extensions of the system. Finally, we believe EPLAN to be a good prototype for other high technology application packages in APL, and hope that it will be viewed in such a light by some of our colleagues in the field.

2. Design of the System

A conscious effort was made to keep the design of the system simple by taking fullest advantage, wherever possible, of the inherent power of APL. As a consequence, the main effort could be devoted to the inherent elements and relationships of econometrics:

- economic variables (time-series, cross-sectional data)
- operations on and with economic variables
- estimation of functional relationships (regression)
- solution of nonlinear economic systems.

By contrast, the development of other econometric systems has been known to require great, sometimes overwhelming, expenditure of effort towards putting in place data manipulation capabilities which are essentially intrinsic to APL. And this is usuallv so even without full interactivity, which we feel is essential if the forecasting and case study capabilities of econometrics are to be brought within the grasp of the analyst and businessman in commerce and government.

We believe that it is desirable to bring similar design considerations to bear on other fields of engineering, applied science and applied mathematics, with the end effect of putting powerful tools where they belong, namely into the realm of practical and useful application by the single practitioner in industry. And we believe that APL is the development aid which is required if the development cost is to be kept within realistic bounds.

3. Economic Variables (see appendix 1)

The basic elements of econometrics, indeed of economics and business, are time series of equidistant data, i.e. data which are equispaced in time, with given "periodicity" and starting period (for example: 1,2,4,12,52 correspond to yearly, semi-annually, quarterly, monthly, weekly data, respectively). (Cross-sectional series can also be handled by EPLAN but we shall not dwell on them here).

In the system a time series is a named vector whose first element is a unique number (computed from the "header") which defines the origin and periodicity, the other "coefficients" being the time series values, themselves.

System functions DF and EC permit the transition between the "real object" and its APL representation:

```
      COST←12 1973 5 DF 4 6 2 7 1

      COST
2368112 4 6 2 7 1

      EC COST[1]
12 1973 5
```

Multi-dimensional time series are multi-dimensional aggregates of vectors, all having the same header.

Display and tabulate functions are available for "pleasing" representation of the time series (see appendix 2 for some examples). The system provides for the insertion of user-constructed headers and row names. Plot functions are available for simultaneous graphic display of several time series.

4. Operations on and between Time Series

The system contains a substantial number of operators, summarized in Table 1. Many of these operators are analogous to the APL primitives, except for special treatment of the header.

TABLE 1: Summary of operators defined in the APL ECONOMETRIC PLANNING LANGUAGE

	Operation	System Function Name
Dyadic	+	P
	-	M
	×	T
	÷	D
	*	PW
	<	L
	≤	LE
	=	E
	≥	GE
	>	G
	≠	NE
	∧	AND
	∨	OR
Monadic	log	LOG
	exp	EXP
	sin	SIN
	cos	COS
	tan	TAN
	abs value	ABS
	cumul value	CUM
	~	NOT
Spec. Oper.	shifts	LAG
	first diff.	DEL
	first quot.	RTO
	rect. distr.	RECT
	norm.distr.	NORM
	change period.	CHANGE

But there are also special operators of unique significance for economic data.

Much of the time the operators leave the headers unchanged, the operations being performed element by element on the coefficients of the time series. However, the special operators often involve a shift in the time series origin, thus affecting both coefficients and headers.

As almost always with APL, by far the easiest way of learning the operations is to sit down at the terminal and to try them out for simple examples. See appendix 2 for some illustrations.

Dyadic operators may involve a scalar left argument, as in

```
      2 T COST
2368112 8 12 4 14 2

      10 D COST
2368112 2.5 1.666666667 5 1.428571429 10
```

or, more frequently, time series for both arguments (as in the addition of time series, for example).

Logical operations on time series produce Boolean time series, which are used in turn to operate on other time series to produce logical effects, such as selection of designated coefficients.

```
      COST T COST G 5
2368112 0 6 0 7 0
```

When an operation involves two time series, such as (unit) cost and volume below, the time frame over which an operation is to be carried out, must be specified explicitly or is determined by default (see appendix 2).

```
      VOLUME←12 1972 11 DF 10 11 11 9 8 10 12 13

      DISPLAY 'VOLUME T COST'

 VARIABLE VOLUME T COST

 PERIODICITY = 12
 ORIGIN = 1973 5
 NO OF ENTRIES = 2

    TIME        VALUE         TIME        VALUE

   1973    5   48.00000  1973    6   78.00000
```

5. Functional Relationships

The economic relationships can be specified by "definitorial equations", and by "structural equations" (to be estimated).

5.1 Definitorial Equations

Econometric models often contain definitorial equations which can be viewed as mere variable transformations. In EPLAN all equations are stored as APL-character vectors:

```
      REVEQU←'DREV←DEL LOG VOLUME T COST'

      REVEQU
DREV←DEL LOG VOLUME T COST
```

The specification ("equation") under the quotes can be executed by means of the APL primitive ⍎ , provided that its right hand side part is completely specified. For example:

```
      COST←12 1972 11 DF 3 2 2 1 4 5 4 6 2 7 1

      ⍎REVEQU

      DREV
2.368E6 ¯0.3102 0 ¯0.8938 1.269 0.4463 ¯0.04082 0.4855
```

5.2 Structural (Stochastic) Equations

Most equations in econometrics contain unknown parameters to be determined ("estimated") from the given data by regression techniques. EPLAN provides a variety of regression options which can be combined in many ways during the estimation procedure.

The following examples have to do with a model which describes interrelations between sales and advertising expenditures for filter and non-filter cigarettes in the U.S. cigarette industry (see the program description manual of EPLAN).

Equations are estimated one by one. They are generally nonlinear in the variables, but are assumed to be linear in the unknown parameters B0, B1, B2 below.

```
    MEQU3←'W1←B0 P (B1 T S1 D S2) P B2 T 1 LAG S1 D S2'
```

```
    INVAR←'1,1 LAG S1,1 LAG S2,1 LAG S1 D S2,DI'
```

(1, LAG X defines a lag of one period in the time series X). An auxiliary variable INVAR is defined to be used in an invocation of the "instrumental variable technique" during regression. The regression technique phase is initiated by the function REGRESS and is characterized by a sequence of answers to "WITH:" requests of the regression function.

```
          MEQU3←'W1' REGRESS '1,S1 D S2,1 LAG S1 D S2'
      WITH: A1 ⍳0
      RHO: ¯0.1732689385

      WITH: INVAR INST 'S1 D S2'
      WITH:
      COEF/VALUE/ST ERR/T-STAT.....

       1    0.30444   0.05852   5.20197
       2    0.33337   0.26026   1.28091
       3   ¯0.14827   0.21698  ¯0.68333

      NO OF VARIABLES.........     2.00000
      NO OF OBSERVATIONS......    10.00000
      SS DUE TO REGRESSION....     0.03541
      SS DUE TO RESIDUALS.....     0.00508
      F-STATISTIC.............    24.38778
      STANDARD ERROR..........     0.02694
      R*2 -STATISTIC..........     0.87450
      R*2 CORRECTED...........     0.83864
      DURBIN WATSON STATISTIC.     1.98565
```

As is shown in the example, the user may invoke a first-order autoregressive correction of the error term by the request. A1 ⍳0, followed by an instrumental variables substitution for the variable S1 D S2 with the "instruments" which are the left hand argument in... INST... . (For a description , see Theil, 1971). A void input (hitting of RETURN at the terminal) to the request WITH: terminates the selection of regression techniques. The techniques are executed in order of selection. At the end, an ordinary least square operation is performed on the observation data modified as indicated by previous requests. Ordinary least squares alone is initiated by a void input to the first WITH: request.

The explicit result of REGRESS is a well-defined estimated equation, i.e. an equation with numerical coefficients, of the same form as a definitorial equation. The totality of definitorial and estimated structural equations will finally constitute a complete econometric model.

```
      MEQU3
W1← ( 0.30444 T 1 ) P ( 0.33337 T S1 D S2 ) P ( ¯0.14827 T 1
       LAG S1 D S2)
```

6. MODELS

6.1 Definition

Models are a set of definitorial and structural equations. A complete model has as many dependent (endogenous) variables as there are equations. In EPLAN a model is represented by an APL-character matrix, where each row corresponds to one equation. The model may be assembled by means of the function DMODEL

```
      EQULIST←'EQU1,MEQU2,MEQU3,EQU4,D1,D2,D3,D4,D5'

      MODEL1←DMODEL EQULIST

      MODEL1

S1← ( 227.177 T WR ) P ( 0.546535 T SL1) P ( 0.120248 T DI )
S2← ( ¯87.0728 T WR ) P ( 0.829077 T SL2) P ( 0.121466 T DI )
W1← ( 0.304443 T 1 ) P ( 0.333374 T SR ) P ( ¯0.148266 T SRL)
W2←(0.385065 T 1) P (¯.291008 T SR) P (.198681 T SRL)
SR←S1 D S2
SL1←1 LAG S1
SL2←1 LAG S2
SRL←SL1 D SL2
WR←W1 D W2
```

The right argument is a list of equation names; the sequence of these names indicates the sequence of the equations in the model (the example is again taken from the manual). The equations have to be written in normalized form, so that each endogenous variable occurs in exactly one equation of the model on the left-hand side. The same variable may not occur on the right-hand side of that equation, except in lagged form. However, it may occur on the right-hand side of other equations.

6.2 Solution

Models are solved in EPLAN by means of a multi-dimensional variant of the well-known GAUSS- SEIDEL technique. Essentially, this technique successively substitutes estimates for the endogenous variables into the right-hand side of the model, starting from "good" initial estimates (see e.g. Ortega and Rheinboldt, 1970). Note, that our model representation in character matrix form can be interpreted as the canonical representation of a valid APL-function. Thus, each iteration of the GAUSS-SEIDEL technique corresponds to one execution of that function. The solution process via the function SOLVE makes direct use of that fact.

Initially, it is usually desirable to reorder the sequence of equations so as to obtain an almost recursive model. This is done by the function ORDER, using VAN DER GIESSEN's substitution algorithm (1970). Convergence may be considerably accelerated by that procedure. Before executing SOLVE, initial estimates for only those endogenous variables must be supplied which are not recursively evaluated within the first iteration (in the example below S1 and S2 as indicated by the execution of ORDER). Also various convergence parameters may be set (not shown). After successful execution of SOLVE the solutions can be called by name.

```
      MODEL2←' 'ORDER MODEL1
SEQUENCE OF EQUS:
SL1,SR ,SL2,SRL,W1 ,W2 ,WR ,S1 ,S2

GIVE INIT APPR FOR:
S1 ,S2

      SALESMODORD
SL1←1 LAG S1
SR←S1 D S2
SL2←1 LAG S2
SRL←SL1 D SL2
W1← ( 0.304443 T 1 ) P ( 0.333374 T SR ) P ( ¯0.148266 T SRL)
W2←(0.385065 T 1) P (¯.291008 T SR) P (.198681 T SRL)
WR←W1 D W2
S1← ( 227.177 T WR ) P ( 0.546535 T SL1) P ( 0.120248 T DI )
S2← ( ¯87.0728 T WR ) P ( 0.829077 T SL2) P ( 0.121466 T DI )



      ⍝ INITIAL ESTIMATES FOR S1 AND S2:

      S1
195401 250.328 398.658 690.081 952.423 1232.111 1414.718
       1526.195 1571.303 1613.578 1657.487 1731.53 1672.207
       1702.119

      S2
195401 3196.318 2856.406 2668.824 2453.522 2178.211 2008.075
       2006.827 1991.59 1971.696 1900.684 1808.007 1590.523
       1633.202
```

```
      'S1,S2,W1,W2' SOLVE MODEL2 *
CONV AT IT: 9

      ⍝ SOLUTIONS, E.G.

      S1
195401 250.328 711.40429 1001.2352 1189.0217 1302.9291 1372.3554
      1449.049 1521.7438 1587.4465 1653.2687 1709.2758 1764.6163
      1818.5691

      S2
195401 3196.318 2870.6459 2612.7354 2405.3939 2233.213 2088.7515
      1978.954 1873.5413 1776.1248 1711.8389 1663.6987 1648.1199
      1651.1821
```

* A revised version of EPLAN requires initial estimates for exogenous variables only, with their names as left arguments of SOLVE (see appendix 2).

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1. Time Series and Model Language

Time Series: Deterministic or random data which may change and can be measured at different points in time. In economics and business planning equidistant time series are mostly used, which are defined at equidistant points in time (periods), e.g. yearly, quarterly, monthly series. The time characteristics of an equidistant time series can be fully described by the periodicity, i.e. the number of periods per year (12 = monthly series) and by the origin, i.e. the period of the "oldest" information (1975 2 = February 1975 for a monthly series or second quarter 1975 for a quarterly series). The period values of the series are thereby ordered with increasing time. A header in EPLAN gives compound information on periodicity and origin (12 1975 2 for a monthly series starting February 1975, 4 1975 2 for a quarterly series starting with the second quarter 1975). Internally, this header is transformed into a unique single number.

Cross-sectional Data: Deterministic or random data which are time independent or measured at the same point in time, e.g. a year's product sales in Belgium, France, UK, Germany, Italy, etc. They are defined in EPLAN by a zero header.

Intersection: of two or more time series: set of values for these series which refer to common periods.

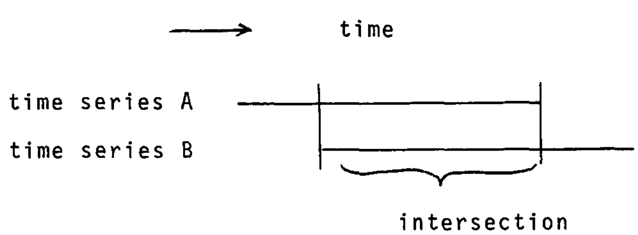


Figure 1.1

Figure 1.1 shows the largest possible intersection for series A and B, which is calculated by EPLAN if no explicit intersection control is assumed (o ST 0). Via explicit intersection control (e.g. 4 1973 1 ST 4 1974 4) the intersection can be reduced further (figure 1.2)

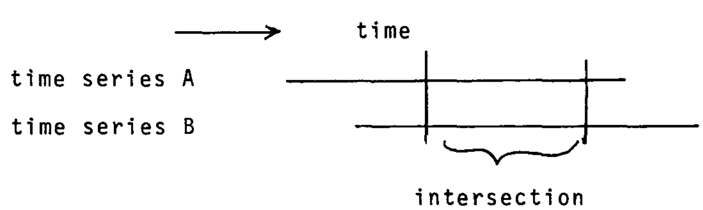


Figure 1.2

The intersection process is used in various situations, e.g. in the application of a dyadic operation such as "Plus" A P B, or in the selection of periods for regression and model solution.

Special Operators: Operators with particular use in econometric modeling, for instance

lags and leads: shifts of periods in a time series (LAG)

first difference: a period's value minus the previous period's value (DEL)

first quotient: a period's value divided by previous period's value (RTO)

periodicity change: aggregation or disaggregation of time series with respect to time, e.g. transformation of a monthly series into a yearly series by averaging, or vice versa by linear interpolation (CHANGE)

rectangular distribution: generation of random variates with uniform distribution, zero mean and standard deviation one (NORM).

Endogenous Variable:

Variable evaluated by the model during solution phase (dependent variable)

Exogenous Variable:

Variable provided by the user before starting the model solution phase. Independent variable supplied from external sources. (policy variables).

2. Regression

Simple Correlation Coefficient: A measure for the correlation between two variables (time series). A value of +1 or -1 signals exact linear dependence (with probability one), a zero value the lack of correlation. Values close to +1 or -1 are usually interpreted as an indication for correlation (though no precise theory is available).

Ordinary Least Squares: The model has the following properties:

a) it is linear in the parameters $b_0, b_1, \ldots, b_n$:

$$y_t = b_0 + b_1 \cdot x_{1t} + b_2 \cdot x_{2t} + \ldots + b_n \cdot x_{nt} + u_t \quad ;$$

(the constant term b_0 may be suppressed)

b) the series $x_{1t}, x_{2t}, \ldots, x_{nt}$ are not collinear, i.e. no series can be expressed as a linear combination of the others; each series varies over time with certain limit properties,

c) the series $x_{1t}, x_{2t}, \ldots, x_{nt}$ are deterministic, all random effects can be aggregated into a residual variable u_t ,

d) The residuals are normally distributed with zero expectation and unknown standard deviation $\sigma \geq 0$, which is independent of time (homoscedasticity),

e) there is no correlation between residuals of different periods (absence of autocorrelation)

The unknown parameters $b_c, b_1, b_2, \ldots, b_n$ and the standard deviation σ are estimated such that the sum of the squared residuals $\sum u_t^2$ becomes minimal (therefore the name "least squares" technique).

Regression Statistics:

a) Parameter Estimates:

STandard ERRor: Indicates accuracy of parameter estimate, e.g. estimate $\pm$ x

ST.ERR = interval within which true parameter value lies with 95.5 percent probability.
T-STATistic: Serves as a test for the hypothesis that the true parameter value is zero, i.e. term may be excluded from the equation. Hypothesis is rejected if absolute value of t is significantly large, typically greater than 1.73 (with 5 percent error in rejecting the true hypothesis that the parameter value is zero, indeed).
Covariance Matrix of parameter estimates (global variable B after regression): Gives further information on the statistical correlation between parameter estimates. Matrix is needed for calculation of other statistics, but may also be used for exploring the estimation accuracy.

b) Total Regression:

F-STATISTIC: Tests the hypothesis that all parameters associated with a real variable (excluding the constant parameter b_c) are simultaneously zero, i.e. the regression reduces to the constant b_o. The hypothesis is rejected if F has a significantly large value (threshold value can be looked up in F-tables with n, T-n "degrees of freedom", n being the number of tested parameters, T the number of observations). A typical value for a three-variables equation would be $F > 8.6$ (assuming 5 percent probability of rejecting a true hypothesis).
NO OF VARIABLES corresponds with number of parameters included in F-test, n.
NO OF OBSERVATIONS, Sum of Squares DUE TO REGRESSION, Sum of Squares DUE TO RESIDUALS are used for calculation of F and other statistics.

STANDARD ERROR is an estimate of the standard deviation of the residuals, see ordinary least squares, and may be used for further judgment of the quality of regression which is expected to produce a relatively small standard error. Note, that the size of the standard error alone gives no indication of the quality of forecasts from the regressed equation, since the errors in the parameter estimates add to uncertainty.

R2-STATISTIC (R2 should be read r-square) is a descriptive measure (similar to a simple correlation coefficient) of the correlation between the lefthand-side y_t and the right hand side $\hat{y}_t = b_0 + b_1 x_{1t} + \cdots + b_n \cdot x_{nt}$ (regressed values of the dependent variable). R2=1, if and only if the observations coincide with the regressed values i.e. there is an exact linear relationship with zero residuals. Otherwise (as in all non-trivial applications) R2 lies between 0 and 1. Here, one associates a high R2-value with high "goodness of fit" of the regressed equation. The R2-statistic is increased with the addition of any further variable to the regression equation

R2 CORRECTED has the same interpretation. A slight correction involving "degrees of freedom" (i.e. number of variables, observations) is applied resulting in a small reduction compared to the uncorrected R2-value above (if R2 is less than 1). It may happen, that R2 corrected becomes negative, whereupon it should be interpreted as zero.

DURBIN WATSON STATISTIC: Tests whether there is no autocorrelation (of order one, see below) between residuals of successive periods (see assumption e for ordinary Least Squares). A value close to 2 (e.g. between 1.65 and 2.35 for a three-variable equation) should be expected. Below and above these threshold points there is an interval of inconclusiveness, while for values beyond the thresholds the hypothesis of no autocorrelation is rejected.

RESIDUALS: Estimates of the residuals can be called in EPLAN by the global variable U after completion of regression.

Generalized Least Squares:

The assumptions of homoscedasticity and autocorrelation (see ordinary Least Squares) may be violated. This defect can be corrected if the covariance matrix x of the residuals is known in advance (which is seldom the case). A special and relatively simple, but important case is the assumption of first-order autocorrelated (autoregressive) residuals.

First-order Autocorrelated (Autoregressive) Residuals: It is assumed that the no-autocorrelation requirement of ordinary Least Squares is violated such that

$$u_t = \rho \cdot u_{t-1} + v_t \qquad (-1 < \rho < +1)$$

with coefficient of autocorrelation ρ (RHo) and v_t being "well-behaved" residuals, i.e. fulfilling all the assumptions of ordinary Least Squares. Such a situation occurs quite frequently in econometric modeling, especially when dynamic feedback effects are not expressed in the equation (e.g. sales depending on sales of previous period). First-order autocorrelation is detected by a poor Durbin-Watson-Statistic in ordinary Least Squares. It may be corrected by either supplying a known value ρ to the EPLAN routine A 1 or, more frequently, by letting the routine calculate an estimate for ρ. Afterwards, the Durbin-Watson-Statistic should be defintely improved. Otherwise, a more complex autocorrelation scheme may distort the data; this can be handled by EPLAN only if the complete convariance matrix is known (via generalized Least Squares).

Instrumental Variables Substitution: This technique may be applied to multi-equation models with high interdependence between endogenous variables. For such models, one major assumption of ordinary Least Squares is violated, namely that all major random effects be expressed through the residuals. Endogenous variables of stochastic nature can now appear on the righthand-side of some equations and may be correlated with the residuals of that equation. In the latter case, the quality of the parameter estimates is distorted such that the estimates do not "converge" to their true values (as should definitely be expected from a good estimation technique) even with increasing number of observations. The estimation technique is said to produce non-consistent estimates. This deficiency can be relaxed by a two-stage procedure which first regresses the "problem" variables on a set of "auxiliary variables", called instrumental variables, and then substitutes the obtained regression results into the original regression.

Typically, the instrumental variables are taken from the set of all predetermined variables in the model, which are the exogenous and the lagged endogenous variables.

Two-stage Least Squares: Instrumental Variable Substitution, where all predetermined variables of the model are used as instruments. In this case it can be proven that the parameter estimates are consistent.

Principal Components Substitution: Has the same objective as Instrumental Variables Substitution. If many instruments are taken, problems of collinearity arise, i.e. various instruments may be almost linearly dependent and thus jeopardize the first-stage regression; see assumption b of ordinary Least Squares. In this case, a specific transformation process is applied to the instruments, before first-stage regression takes place. The results from this transformation process are called principal components. The procedure further permits the user to reduce the number of transformed instruments in a systematic fashion, since the principal components are ordered by decreasing variance and, therefore influence, in explaining the variance of the regressed endogenous variable. A maximum amount of total variance (of all instruments) may be specified in advance; a typical value is 90 - 95 percent.

Polynomial Distributed Lags: Frequently it is suggested that the left-hand-side variable depends on subsequent lags of one variable (besides further variables on the right-hand-side), i.e.

$$y_t = b_0 + \underbrace{b_1 \cdot x_{1t} + b_2 \cdot x_{1,t-1} + \ldots + b_l \cdot x_{1,t-l+1}}_{\text{distributed lags}} + b_{l+1} \cdot x_{2,t} + \ldots$$

If many lags are included, problems of collinearity as well as number of available observations arise. A popular way out assumes a-priori restrictions on the "weights" ($b_1, b_2, \ldots, b_l$) namely that they lie on a polynomial. Through this assumption, the above problems can be reduced, but only at the cost of a parameter distortion (due to the interpolation by a polynomial of a degree which should be selected smaller than the number of weights). The interpolation of parameters by polynomials has been suggested by ALMON.

3. Model Solution

Simulation: The model is solved for its endogenous variables over a given number of periods (simulation horizon) assuming known values for the exogenous variables.

Gauss-Seidel Solution Technique: Initial guesses (solution estimates) for the endogenous variables (solutions) are provided. These guesses are "improved" through successive substitution into the model, until a solution has been found (i.e. the substitution process has converged, see convergence criterion). Each model evaluation with a set of solution estimates corresponds to one "iteration".

New estimates are immediately used in subsequent equations, even during the same iteration. This solution technique may not converge in general, but does often for econometric models (see recursive models, below).

Convergence Criterion serves for the detection of the solution. In EPLAN the solution process has converged if for all endogenous variables the relative difference between two subsequent iterations is less than a predefined value (set by SOLVECON. A typical value is 0.005 = 5 percent deviation or 0.001 = 1 percent relative deviation.)

Iteration Count: Maximum number of iterations before stopping an unsuccessful process (set by SOLVEIT).

Relaxation Parameter: A value α with $0 < \alpha \leq 1$ which effects a damping if the solution process oscillates, and in doing so speeds up conversion (reduces number of required iterations). The parameter should be set to 1, if no oscillation is assumed. It may be redefined during the solution process by interrupting the SOLVE function (the relaxation parameter is defined by SOLVEREL).

Recursive Models: There is no mutual interdependence between endogenous variables, but all endogenous variables are completely specified by the exogenous variables and endogenous variables from previous equations in the model (causal chain). In this case the solution process is trivial and terminates after the first iteration.

Reordering of Model Equations: Recursiveness of models can be hidden by the sequence of the equations in the model. Often, by reordering the equations, a model form can be arrived at which is "almost" recursive, i.e. there is mutual interdependence only for a few variables. Solving the model in this form can improve convergence speed considerably. EPLAN uses VAN DER GIESSEN's reordering algorithm.

4. Suggested Literature:

Koutsoyiannis, A., Theory of Econometrics, Macmillan, London 1973

(quite complete and detailed treatment of regression techniques with examples and easy-to-reach mathematical notation)

Theil, H., Principles of Econometrics, J. Wiley, New York 1971

(more mathematically oriented treatment of regression techniques, including proof of theorems).

Van der Giessen, A.A., Solving Non-linear Systems by Computer: A New Method, in: Statistica Neerlandica 24 (1970) p.1-10

(treats Gauss-Seidel solution technique including reordering of equations).

APPENDIX 2

```
⍝ DEMONSTRATION OF EPLAN

⍝A DESIGNATE APL COMMENTS
⍝E DESIGNATE EPLAN COMMENTS

⍝A )LOAD 'NAME' ... LOADS A WORKSPACE

⍝E EPLAN IS USUALLY IN TWO WORKSPACES ... EPL1, EPL2
⍝E (COMBINE INTO ONE WORKSPACE EPLAN, WHEN POSSIBLE)

      )LOAD EPL1
SAVED  11.42.06 06/08/76
PROGRAM NO.5796-PDW. COPYRIGHT IBM CORP. 1974. REFER TO INSTRUCTI
ONS ON COPYRIGHT NOTE FORM NO. 120-2083

⍝A CHANGE THE WORKSPACE NAME TO  EPLANDEMO

      )WSID EPLANDEMO
WAS EPL1

      )WSID
EPLANDEMO

⍝A SAVE THE WORKSPACE UNDER ITS NEW NAME

      )SAVE
WS NAME IS TOO LONG

      )WSID EPLANDEM
WAS EPLANDEMO
      )SAVE
  11.51.17 06/08/76 EPLANDEM

⍝A EXHIBIT THE GROUPS OF FUNCTIONS IN THE WORKSPACE

      )GRPS
BASEGRP DISPLAYGRP      PLOTGRP

      )LOAD EPL2
SAVED  11.41.33 06/08/76
PROGRAM NO.5796-PDW. COPYRIGHT IBM CORP. 1974. REFER TO INSTRUCTI
ONS ON COPYRIGHT NOTE FORM NO. 120-2083

      )GRPS
ADDENDUM        BASEGRP FILEGRP FILEGRPC        REGRESSGRP
SOLVEGRP        TRANSLATEGRP

      )LOAD EPLANDEM
SAVED  11.51.17 06/08/76
PROGRAM NO.5796-PDW. COPYRIGHT IBM CORP. 1974. REFER TO INSTRUCTI
ONS ON COPYRIGHT NOTE FORM NO. 120-2083
```

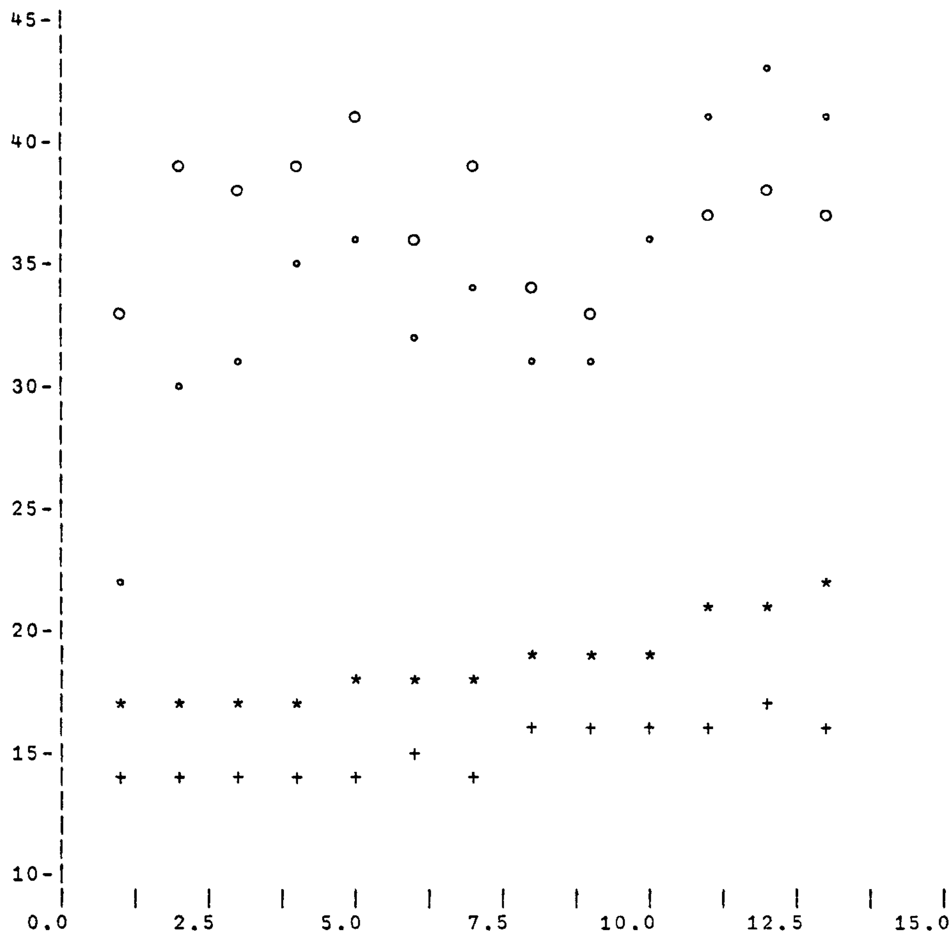
```
      ⍝A IF OTHER GROUPS NEEDED FROM EPL2, USE  COPY COMMAND

      )COPY EPL2 REGRESSGRP
SAVED  11.41.33 06/08/76

      )WSID
EPLANDEM

      )GRPS
BASEGRP DISPLAYGRP      PLOTGRP REGRESSGRP

      )SAVE
  13.31.55 06/08/76 EPLANDEM

      ⍝A WHEN GROUP NO LONGER NEEDED, DELETE IT BY ERASE COMMAND

      )ERASE REGRESSGRP

      )GRPS
BASEGRP DISPLAYGRP      PLOTGRP



      ⍝A APL MAY INTERRUPT PROCESSING FOR A NUMER OF REASONS
      ⍝A MOST OFTEN THE INTERRUPTS WILL BE CAUSED BY USER ERROR
      ⍝A OCCASIONALLY THERE MAY BE RESOURCE LIMITATIONS

      ⍝A OCCASIONALLY AN INTERRUPT WILL BE USER GENERATED (ATTN KEY)
      ⍝  ************************************************************

      ⍝A FOR SOME SUGGESTED ACTIONS YOU MAY WISH TO TURN TO PAGE 14
```

```
      ⍝A AN APL OBJECT (NUMBER, CHARACTER, VECTOR, MATRIX)
      ⍝A CAN BE SPECIFIED (ASSIGNED A VALUE) BY THE OPERATOR ←

      A1← 3 5 6

      A1
3 5 6

      ⍝A THE VECTOR  A1 CAN BE EXTENDED BY CATENATION...USE OF COMMA

      A1←A1,9 7

      A1
3 5 6 9 7


      ⍝E A BABY FOOD MODEL
      ⍝  ≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠

      ⍝E EPLAN DEALS WITH TIME SERIES, I.E. DATA OVER A TIME PERIOD

      ⍝E EXAMPLE ... BABIES FROM THIRD QUARTER 1960 ONWARD
      ⍝E QUARTERLY DEMAND, PRICE, UNITCOST FOR (OF) BABYFOOD

      ⍝ AN ECONOMETRIC MODEL STUDIES THE RELATIONSHIPS BETWEEN
      ⍝                     THESE QUANTITIES
      ⍝ IT WILL BE ESTIMATED (CALIBRATED) FROM PAST DATA
      ⍝ AND USED TO FORECAST DEMAND AND PRICE FOR FUTURE PERIODS


      ⍝E DATA INPUT
      ⍝  ≠≠≠≠≠≠≠≠≠≠

      BABIES←22 30 31 35 36 32 34 31 31 36 41 43 41
      DEMAND←33 39 38 39 41 36 39 34 33 36 37 38 37

      ⍝E IN THE SYSTEM, THE TIME SERIES ARE VECTORS WITH A HEADER
      ⍝E FOLLOWED BY THE DATA, THE HEADER  4 1960 3  SIGNIFIES
      ⍝E START OF THE TIME SERIES IN THE THIRD QUARTER OF 1960

      BABIES←4 1960 3 DE BABIES

      BABIES
784304 22 30 31 35 36 32 34 31 31 36 41 43 41

      ⍝E THE DECODE FUNCTION DE (ALL SYSTEMS FUNCTIONS USE
      ⍝E UNDERSTRUCK CHARACTERS) CHANGES THE HEADER TO ONE NUMER

      DEMAND←4 1960 3 DE DEMAND

      DEMAND
784304 33 39 38 39 41 36 39 34 33 36 37 38 37
```

```
      PRICE←4 1960 3 DF 16.6 16.6 17.1 17.3 17.5 17.5

      PRICE←PRICE,17.9 18.5 18.5 19 20.6 20.6 21.6

      PRICE
784304 16.6 16.6 17.1 17.3 17.5 17.5 17.9 18.5 18.5 19 20.6 20.6
21.6

      UNITC←4 1960 3 DF 14.4 13.7 14.1 14.2 14.1 14.8 14.2 15.5 15.7

      UNITC←UNITC,15.6 16.45 16.5 16.25

      DISPLAY BA
              v
              'BABIES'

VARIABLE BABIES


PERIODICITY = 4

ORIGIN = 1960 3

NO OF ENTRIES = 13


    TIME        VALUE         TIME        VALUE

  1960    3   22.00000   1962    2   31.00000
  1960    4   30.00000   1962    3   31.00000
  1961    1   31.00000   1962    4   36.00000
  1961    2   35.00000   1963    1   41.00000
  1961    3   36.00000   1963    2   43.00000
  1961    4   32.00000   1963    3   41.00000
  1962    1   34.00000

      DISPLAY 'PRICE'

VARIABLE PRICE


PERIODICITY = 4

ORIGIN = 1960 3

NO OF ENTRIES = 13


    TIME        VALUE         TIME        VALUE

  1960    3   16.60000   1962    2   18.50000
  1960    4   16.60000   1962    3   18.50000
  1961    1   17.10000   1962    4   19.00000
  1961    2   17.30000   1963    1   20.60000
  1961    3   17.50000   1963    2   20.60000
  1961    4   17.50000   1963    3   21.60000
  1962    1   17.90000
```

```
      10 5 9 3 TABULATE 'DEMAND,PRICE,,BABIES,UNITC,,VOL←DEMAND T PRICE'

               1960 3   1960 4   1961 1   1961 2   1961 3

DEMAND         33.000   39.000   38.000   39.000   41.000
PRICE          16.600   16.600   17.100   17.300   17.500

BABIES         22.000   30.000   31.000   35.000   36.000
UNITC          14.400   13.700   14.100   14.200   14.100

VOL←DEMAND T PRICE  547.800  647.400  649.800  674.700  717.500

               1961 4   1962 1   1962 2   1962 3   1962 4

DEMAND         36.000   39.000   34.000   33.000   36.000
PRICE          17.500   17.900   18.500   18.500   19.000

BABIES         32.000   34.000   31.000   31.000   36.000
UNITC          14.800   14.200   15.500   15.700   15.600

VOL←DEMAND T PRICE  630.000  698.100  629.000  610.500  684.000

               1963 1   1963 2   1963 3

DEMAND         37.000   38.000   37.000
PRICE          20.600   20.600   21.600

BABIES         41.000   43.000   41.000
UNITC          16.450   16.500   16.250

VOL←DEMAND T PRICE  762.200  782.800  799.200
```

50 50 2 PLOT '*DEMAND,PRICE,BABIES,UNITC*'



ABSCISSA = TIME STARTING FROM 1960 3

o = *DEMAND*

* = *PRICE*

∘ = *BABIES*

\+ = *UNITC*

```
⍝ SUPPOSE THE BABYFOOD PEOPLE KNOW THAT
⍝      DEMAND DEPENDS ON (NO. OF) BABIES AND INVERSELY ON PRICE
⍝ AND AT THE SAME TIME
⍝      PRICE DEPENDS ON DEMAND UND UNITCOST
⍝ (A CONSTANT TERM  1  IS OFTEN INCLUDED)
⍝  ≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠

⍝ THE COEFFICIENTS IN THE RELATIONS ARE DETERMINED FROM THE DATA
⍝ BY REGRESSION TECHNIQUES


⍝ WORK WITH 5 DIGIT PRECISION

PRECISION 5

⍝ ANSWER REGRESSION REQUEST  WITH: BY HITTING  RETURN
⍝ (THE SYSTEM PROVIDES MANY OTHER OPTIONS, SUCH AS
⍝       MULTIPLE REGRESSION FEATURES)




      )COPY EPL2 REGRESSGRP
SAVED  11.41.33 06/08/76


      EQ1←'DEMAND' REGRESS 'BABIES, 1 D PRICE, 1'
WITH:


COEF/VALUE/ST ERR/T-STAT.....

 1     0.71825    0.09427     7.61944
 2   766.61721  116.01833     6.60773
 3   ¯29.48305    9.18503    ¯3.20990

NO OF VARIABLES.........   2.00000
NO OF OBSERVATIONS......  13.00000
SS DUE TO REGRESSION....  62.25542
SS DUE TO RESIDUALS.....  10.66766
F-STATISTIC.............  29.17951
STANDARD ERROR..........   1.03284
R*2 -STATISTIC..........   0.85371
R*2 CORRECTED...........   0.82446
DURBIN WATSON STATISTIC.   2.31903


⍝ THE STATISTICS (T-STAT, F-STAT, R*2 - STAT)
⍝ INDICATE THAT THE RESULTS ARE SIGNIFICANT (STANDARD TEXTS)
```

```
      EQ2←'PRICE' REGRESS 'DEMAND,UNITC,1'
WITH:

COEF/VALUE/ST ERR/T-STAT.....

 1     0.21940    0.05673   3.86759
 2     1.68975    0.13992  12.07639
 3   ¯15.10435    3.48257  ¯4.33713

NO OF VARIABLES.........    2.00000
NO OF OBSERVATIONS......   13.00000
SS DUE TO REGRESSION....   29.53114
SS DUE TO RESIDUALS.....    2.01809
F-STATISTIC.............   73.16589
STANDARD ERROR..........    0.44923
R*2 -STATISTIC..........    0.93603
R*2 CORRECTED...........    0.92324
DURBIN WATSON STATISTIC.    1.83016

      ⍝E THE MODEL EQUATIONS CAN BE GROUPED TOGETHER AS A MODEL

      BABYFOODMODEL←DMODEL 'EQ1,EQ2'

      BABYFOODMODEL
DEMAND← ( 0.71825 T BABIES ) P ( 766.62 T 1 D PRICE) P ( ¯29.483
T 1 )
PRICE← ( 0.2194 T DEMAND) P ( 1.6898 T UNITC ) P ( ¯15.104 T 1 )

      ⍝E ERASE THE REGRESSGRP WHICH IS NO LONGER NEEDED
      )ERASE REGRESSGRP

      )SAVE
  14.44.04 06/08/76 EPLANDEM

      ⍝E THERE ARE TRANSLATION AND RETRANSLATION FACILITIES


       )COPY EPL2 ADDENDUM
 SAVED  11.41.33 06/08/76

       ⍝A CHANGE WIDTH FOR BETTER PRINTOUT

       )WIDTH 70
 WAS 65

       RETRANS BABYFOODMODEL
 DEMAND← ( 0.71825 × BABIES ) + ( 766.62 × 1 ÷ PRICE) + ( ¯29.483 × 1 )
 PRICE← ( 0.2194 × DEMAND) + ( 1.6898 × UNITC ) + ( ¯15.104 × 1 )
```

```
      ⍝E FORECASTING REQUIRES ASSUMPTIONS FOR THE
      ⍝E       EXOGENOUS VARIABLES (BABIES, UNITCOST)

      ⍝E FOLLOWED BY THE SOLUTION OF THE ESTIMATED MODEL FOR THE
      ⍝E       ENDOGENOUS VARIABLES (DEMAND, PRICE)

      ⍝E OVER THE FORECASTING HORIZON, SAY 4 QUARTERS STARTING
      ⍝E       IN 4TH QUARTER OF 1963
      ⍝  ≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠

      ⍝E RETAIN ONLY BASEGRP (OTHERWISE SYMBOL TABLE MAY HAVE TO
      ⍝E       BE ENLARGED)

      )ERASE DISPLAYGRP PLOTGRP ADDENDUM

      )COPY EPL2 SOLVEGRP
SAVED  11.41.33 06/08/76

      )GRPS
BASEGRP SOLVEGRP

      ⍝E LET ASSUMPTIONS FOR THE NEXT YEAR BE
      ⍝E          BABIES...40 37 41 42
      ⍝E          UNITC ...16.6 17.1 17.3 17.6
      ⍝E THE FOUR QUARTERLY VALUES FOR DEMAND, PRICE WILL BE COMPUTED
      ⍝E          FROM THE MODEL




      BABIES←BABIES,40 37 41 42
      UNITC←UNITC,16.6 17.1 17.3 17.6

      ⍝A USE 6 DIGITS FOR PRINTING

      )DIGITS 6
WAS 10
      ⍝E SAVE INITIAL DEMAND AND PRICE DATA
      DEMAND0←DEMAND
      PRICE0←PRICE


      BABIES
784304 22 30 31 35 36 32 34 31 31 36 41 43 41 40 37 41 42

      UNITC
784304 14.4 13.7 14.1 14.2 14.1 14.8 14.2 15.5 15.7 15.6 16.45 16.5
16.25 16.6 17.1 17.3 17.6
```

```
      ⍝E BEFORE INVOKING SOLUTION PROCEDURE, SET
      ⍝E   CONVERGENCE, ITERATION, RELAXATION PARAMETERS
      ⍝E SUPPRESS  SOLVE OUTPUT PARTIALLY

      SOLVECON .005
      SOLVEIT 500
      SOLVEREL 1

      OUTPUT 0

      ⍝E SOLVE BABYMODEL FOR DEMAND AND PRICE (WITH EXOGENOUS VARIABLES
      ⍝E BABIES AND UNITC) FOR NEXT FOUR QUARTERS
      ⍝E SET TIME FRAME BY MEANS OF  ST

      4 1963 4 ST 4 1964 3


      'BABIES,UNITC' SOLVE BABYFOODMODEL
PER 4 1963 4 CONVERGED AT ITERATION 5

PER 4 1964 1 CONVERGED AT ITERATION 4

PER 4 1964 2 CONVERGED AT ITERATION 5

PER 4 1964 3 CONVERGED AT ITERATION 4

      ⍝E THE COMPUTATIONS (GAUSS-SEIDEL METHOD) WERE EXECUTED
      ⍝E TIME PERIOD BY TIME PERIOD, AND CONVERGED TO A SOLUTION

      ⍝E THE RESULTS ARE IN  DEMAND AND PRICE

      DEMAND
785604 35.9884 33.3681 35.1052 34.9984

      PRICE
785604 20.8425 21.1125 21.8316 22.3151

      )ERASE SOLVEGRP
      )COPY EPL1 DISPLAYGRP PLOTGRP
INCORRECT COMMAND
      )COPY EPL1 DISPLAYGRP
SAVED  11.42.06 06/08/76

      10 5 9 3 TABULATE 'PRICE,DEMAND'


             1963 4    1964 1    1964 2    1964 3


PRICE        20.843    21.113    21.832    22.315

DEMAND       35.988    33.368    35.105    34.998
```

```
⍝E PLOT NEWLY OBTAINED VALUES
⍝  ≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠

40 50 10 PLOT 'PRICE,DEMAND'

36-*
  -
  -                *
  -                        *
  -
34-
  -
  -        *
  -
  -
32-
  -
  -
  -
  -
30-
  -
  -
  -
  -
28-
  -
  -
  -
  -
26-
  -
  -
  -
  -
24-
  -
  -
  -
  -                        o
22-                o
  -
  -        o
  -o
  -
20-
   |   |   |   |   |   |   |
   1       2       3       4

   ABSCISSA = TIME STARTING FROM 1963 4

   o = PRICE

   * = DEMAND
```

```
      ⍝E USE TABULTEXT FOR TABLES WITH ROW AND COLUMN TEXT
      ⍝  ≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠

      TABHEAD←'VALUES OF TIME SERIES OVER FORECAST '
      TABHEAD←TABHEAD,'HORIZON'
      TEXTDEMAND←'DEMAND'
      TEXTPRICE←'PRICE'
      TEXTBABIES←'BABIES'
      TEXTUNITC←'UNIT-COST'

      )COPY EPL2 TABULTEXT
SAVED  11.41.33 06/08/76


      10 5 9 3 TABULTEXT 'DEMAND,PRICE,,BABIES,UNITC'


VALUES OF TIME SERIES OVER FORECAST HORIZON


             1963 4    1964 1    1964 2    1964 3


DEMAND       35.988    33.368    35.105    34.998

PRICE        20.843    21.113    21.832    22.315

BABIES       40.000    37.000    41.000    42.000

UNIT-COST    16.600    17.100    17.300    17.600

      ⍝E CONSTRUCT DEMAND AND PRICE TIME SERIES FROM THIRD
      ⍝E QUARTER 1960 TO THIRD QUARTER 1964
      ⍝E BY CATENATION OF INITIAL SERIES WITH FINAL SERIES
      ⍝A (THE HEADER OF THE FINAL SERIES BEING DROPPED)

      DEMAND←DEMAND0,1↓DEMAND
      PRICE←PRICE0,1↓PRICE

      DEMAND
784304 33 39 38 39 41 36 39 34 33 36 37 38 37 35.9884 33.3681 35.1052
34.9984
      PRICE
784304 16.6 16.6 17.1 17.3 17.5 17.5 17.9 18.5 18.5 19 20.6 20.6 21.6
20.8425 21.1125 21.8316 22.3151
```

```
⍝E USE THE COMMAND  CHANGE  FOR CHANGING OF PERIODICITIES
⍝E THERE ARE TFREE OPTIONS FOR COMPRESSION
⍝E TEN OPTIONS FOR EXPANDING OF PERIODICITY
⍝  ≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠≠

⍝E COMPRESS TIME SERIES  DEMAND  TO ANNUAL SERIES
⍝E USING OPRION 1...AVERAGE VALUES

DEMA←1 1 CHANGE DEMAND

DISPLAY 'DEMA'

VARIABLE DEMA

PERIODICITY = 1

ORIGIN = 1960 1

NO OF ENTRIES = 5

   TIME        VALUE         TIME        VALUE

  1960    1   36.00000  1963    1   36.99710
  1961    1   38.50000  1964    1   34.49052
  1962    1   35.50000

⍝E CREATE A SEMI+ANNUAL TIME SERIES BY LINEAR INTERPOLATION

DEMSA←2 2 CHANGE DEMA

DISPLAY 'DEMSA'

VARIABLE DEMSA

PERIODICITY = 2

ORIGIN = 1960 1

NO OF ENTRIES = 10

   TIME        VALUE         TIME        VALUE

  1960    1   34.75000  1962    2   35.50000
  1960    2   36.00000  1963    1   36.24855
  1961    1   37.25000  1963    2   36.99710
  1961    2   38.50000  1964    1   35.74381
  1962    1   37.00000  1964    2   34.49052
```

```
      ⍝A POSSIBLE INTERRUPTS OF THE SYSTEM: SUGGESTIONS
      ⍝  ********************************************

      ⍝A SOME ERRORS LEAD TO A MESSAGE, BUT REQUIRE MERELY
      ⍝A A CORRECT RESTATEMENT (IN THE FOLLOWING THE LEFT HAND
      ⍝A ARGUMENT OF A FUNCTION IS MISSING)

      TABULATE 'PRICE'
SYNTAX ERROR
      TABULATE'PRICE'
      ∧
```

```
      ⍝A OTHER ERRORS LEAD TO SUSPENSION OF EXECUTION WITHIN
      ⍝A A FUNCTION, OR SEQUENCE OF FUNCTIONS
      ⍝A ACTION: QUERY WITH )SI, CLEAR WITH : →

      40 60 20 PLOT 'DEMAND'
UNACCEPTABLE WIDTH
DOMAIN ERROR
ARGUMENT[4]  →⎕←B[A-~⍳1;]
              ∧
      )SI
ARGUMENT[4] *
PLOT[153]
MPLOT[3]
PLOT[5]

      →
      )SI
```

```
      ⍝A COPYING MAY LEAD TO WORKSPACE OVERFLOW OR 'SYMBOL TABLE FULL'
      ⍝A ACTION: REMOVE UNNEEDED GROUPS BY  )ERASE
      ⍝A         OR: INCREASE SIZE OF SYMBOL TABLE  IN CLEAR WS)

      ⍝E THE FOLLOWING PERTAINS TO APL/SV ONLY

      )GRPS
BASEGRP DISPLAYGRP      PLOTGRP

      )COPY EPL2 SOLVEGRP REGRESSGRP ADDENDUM
SYMBOL TABLE FULL

      )SAVE
  13.36.50 06/08/76 EPLANDEM
      )CLEAR
CLEAR WS
      )SYMBOLS 500
WAS 256
      )COPY EPLANDEM
SAVED  13.36.50 06/08/76
      )COPY EPL2 SOLVEGRP REGRESSGRP ADDENDUM
SAVED   7.08.31 03/02/76
      )WSID EPLANDEM
WAS CLEAR WS
      )GRPS
ADDENDUM        BASEGRP DISPLAYGRP       PLOTGRP REGRESSGRP       SOLVEGRP
```